

MONTHLY NOTICES
OF THE
ROYAL ASTRONOMICAL SOCIETY

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MEETING OF 1937 JUNE 11

Dr. H. Spencer Jones, President, in the Chair.

The election by the Council of the following Fellows was duly confirmed :—

Dr. Beno Gutenberg, Seismological Laboratory, 220 San Rafael Avenue N., Pasadena, Cal., U.S.A. ; and

Dr. József Wodetzky, Professor of Astronomy, Budapest University, Múzeum-Krt. 6/8, Budapest VIII.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended :—

Arthur P. Rollett, B.Sc., 4 Oak Lane, Sevenoaks, Kent (proposed by G. S. Le Beau).

Seventy-eight presents were announced as having been received since the last meeting, including :—

Harold Jeffreys : Scientific Inference (presented by the author) ;

British National Committee for the Polar Year : British Polar Year Expedition, Fort Rae, 1932-33. Vols. 1, 2 (presented by the Air Ministry).



THE CORES OF CERTAIN FRAUNHOFER LINES.

R. O. Redman, M.A., Ph.D.

Summary

The profiles of $\lambda\lambda$ 4046, 4064, 4072, FeI; 4078, SrII; 4326, FeI; and 4340, $H\gamma$ have been measured in light from undisturbed regions at the centre of the Sun's disc, special attention being directed to the line centres. The corrections on account of finite resolving power have been found by measuring the instrumental profile, using krypton lines. The corrected central intensities are: 4046, 2.1 per cent.; 4064, 1.9 per cent.; 4072, 2.2 per cent.; 4078, 2.0 per cent.; 4326, 2.5 per cent.; $H\gamma$, 14.5 per cent. A detailed discussion of likely sources of error is made, with a description of the precautions taken to avoid them. It is concluded that the probable error of the central intensities should not exceed 0.7 per cent., a figure greater than that indicated by internal evidence alone. A suggestion made in an earlier paper is repeated, namely, that the deviations of the observed central intensities from zero are mostly due to local disturbances in the Sun's atmosphere which are characterized by average line of sight velocities of 1 to 1.5 km./sec.

Although our knowledge concerning the method of formation of the Fraunhofer lines of the solar spectrum has increased enormously during recent years, there are still many points on which both theory and observation speak with some uncertainty. One of these is the question of the residual intensities at the centres of the lines. The first theories, based on rather simplified assumptions, showed that at least the stronger Fraunhofer lines should be almost perfectly black at their centres, the predicted intensity there being well under 1 per cent. of the adjacent continuous spectrum. Confronting this result were observations which claimed to show values of 10 per cent. or more as being the usual rule. This initial contradiction has been softened to some extent; first, it has been realized that what may be called the classical theory of line formation is strictly applicable only to resonance lines and that certain processes, *e.g.* the interlocking of atomic energy levels, are possibly capable of raising central intensities; and secondly, it has equally come to be understood that many of the early observations of central intensities were subject to large systematic errors, so that the values which theory should be called upon to explain may be much lower than formerly supposed.

The technique of accurate photographic photometry is nowadays so well known that the two principal sources of error in observations of this kind are likely to be due to (1) scattered light in the spectrograph, and (2) distortion of the spectrum line by reason of finite resolving power of the optical train

employed. Eighteen months ago a profile of the calcium line $\lambda 4227$ was published,* which was hoped to be free from errors of this kind. The residual intensity was 2 per cent., about the lowest figure which up to then had been deduced for this or any other line. It has been the object of the work about to be described to measure with the greatest possible accuracy the central parts of a few well-known lines in the region $\lambda\lambda 4000-4500$, using essentially the same technique as before, but with a still greater effort to track down and eliminate all conceivable sources of error.

Observations

The McClean spectrograph of the Solar Physics Observatory, Cambridge, has again been used. Several improvements have been made in the instrument: (a) the 30° prism has been refigured, a noticeable improvement in

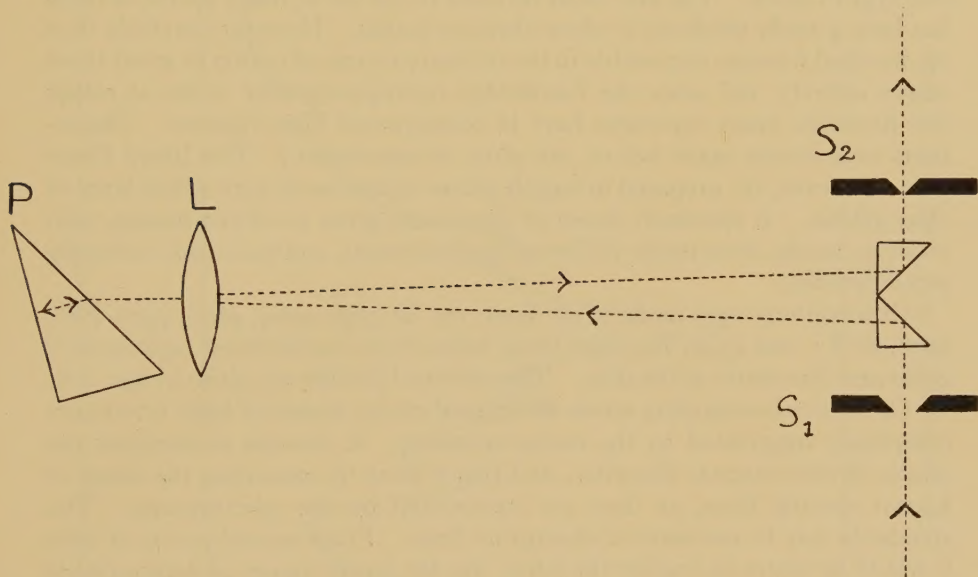


FIG. 1.—Monochromator. S_1 , monochromator slit; L , lens of angular aperture greater than that of main spectrograph; P , 30° prism on rotating table; S_2 , main spectrograph slit.

definition being obtained; (b) a minimum deviation linkwork operated by electric motor has been made for the prism train; (c) a new temperature case, enclosing the whole of the instrument except the slit plate, has been constructed; (d) a simpler monochromator has been devised.

The monochromator is due to Mr. W. Moss, to whom the writer wishes to express his thanks. It is essentially a small Littrow spectrograph and is shown in fig. 1, which is self-explanatory. Adjustment in wave-length is obtained simply by rotating the prism and is much more convenient than with the monochromator formerly used, while there is also the great advantage

* *M.N.*, 95, 742, 1935 (referred to later as I).

that it is now possible to make use of the regular telescope lens of the McClean installation (18.3 metres focal length, solar image about 16 cm. diameter). The earlier work showed that it is safe to admit about 30 Å. of the solar spectrum into the main spectrograph and this is possible with the monochromator slit used very wide, so that across the main spectrograph slit the illumination is uniform except quite near the ends of the region used. On every occasion when photographs were taken a routine inspection of the collimator illumination was made to ensure that the whole spectrograph aperture was uniformly filled.

The procedure with regard to photography, plate calibration, development, use of microphotometer and reduction of microphotometer tracings was the same as before. Backed Ilford Ordinary Process plates were used, since they are known to possess certain advantages to which reference will be made later. In general these plates have specially good characteristics for this type of work. The only fault revealed in the use of many scores of them has been a ready tendency to show abrasion marks. However carefully they are handled it seems impossible in the ordinary course of events to avoid these marks entirely, and since the Cambridge microphotometer works at rather low densities, many exposures have in consequence been rejected. (Rejections were always made before, not after, measurement.) The Ilford Company, however, are prepared to supply plates supercoated with a thin layer of clear gelatin. A specimen dozen of these have given excellent results, with no stress marks, even under deliberate maltreatment, and their use is strongly recommended.

Observations were made of six lines, viz. $\lambda\lambda$ 4046, 4064, 4072, 4326, *FeI*; 4078, *SrII*; and 4340, *H γ* , light being taken from undisturbed regions at or quite near the centre of the disc. The observed profiles are given in figs. 2-7.

Of the two outstanding errors mentioned above, scattered light is virtually completely suppressed by the monochromator. It remains to eliminate the effects of instrumental distortion, and this is done by measuring the shape of known spectral lines, as they are represented by the spectrograph. The standards may be emission or absorption lines. From several points of view it would be better to employ the latter, but the writer knows of none suitable and easily available in the spectrum region under consideration, so that use has been made of emission krypton lines, obtained from an ordinary Geissler tube. In the published investigation on λ 4227, lines of argon and neon were used for this purpose, chiefly the latter, on account of their great intrinsic brightness, but it was necessary to assume that the instrumental profile, *i.e.* the shape of an infinitely narrow line as depicted by the spectrograph, did not vary in linear dimensions on the plate with changing wavelength. The krypton lines have the advantage that they are very much nearer in wave-length to the Fraunhofer lines being measured, while their structure has been very thoroughly examined with interferometers of very high resolving power.

The first two determinations of the instrumental profile had to be rejected. One was found to show a small but distinct loss of definition due to vibration from the thermostat fan, and after this trouble had been cured a second

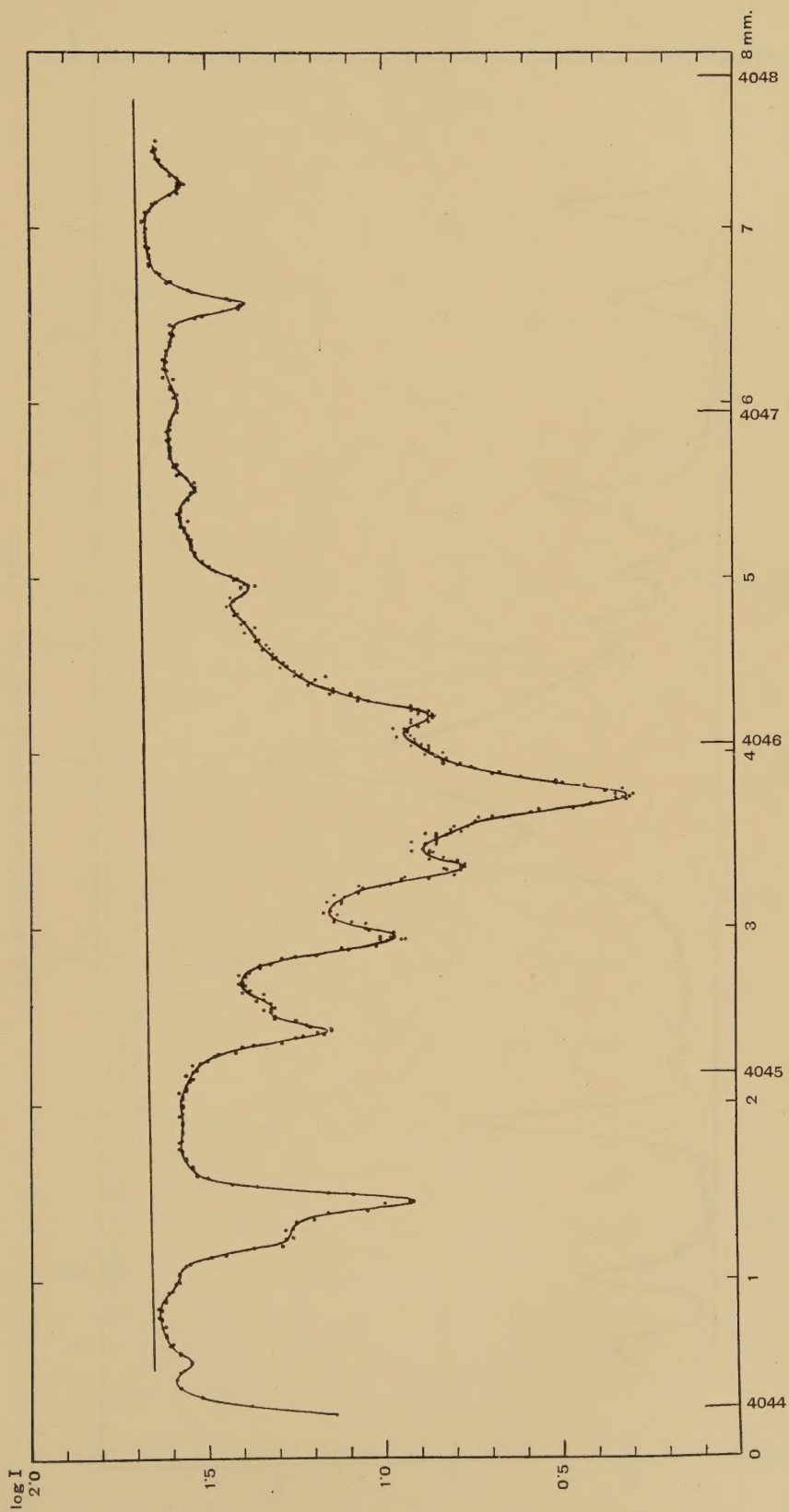


FIG. 2.—Observed profile of λ 4046, Fe I; 8 plates.

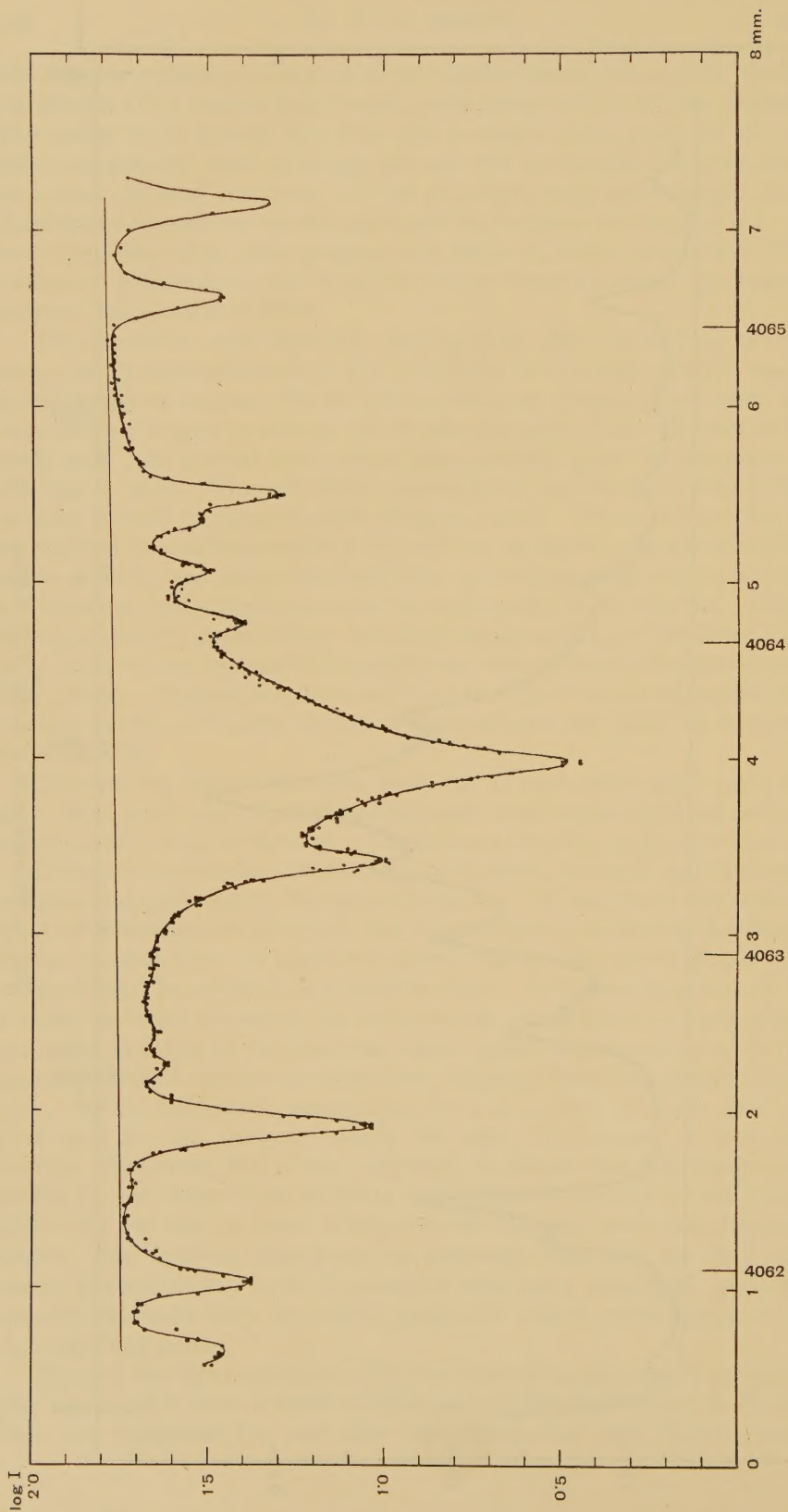


FIG. 3.—Observed profile of λ 4064, Fe I; 11 plates.

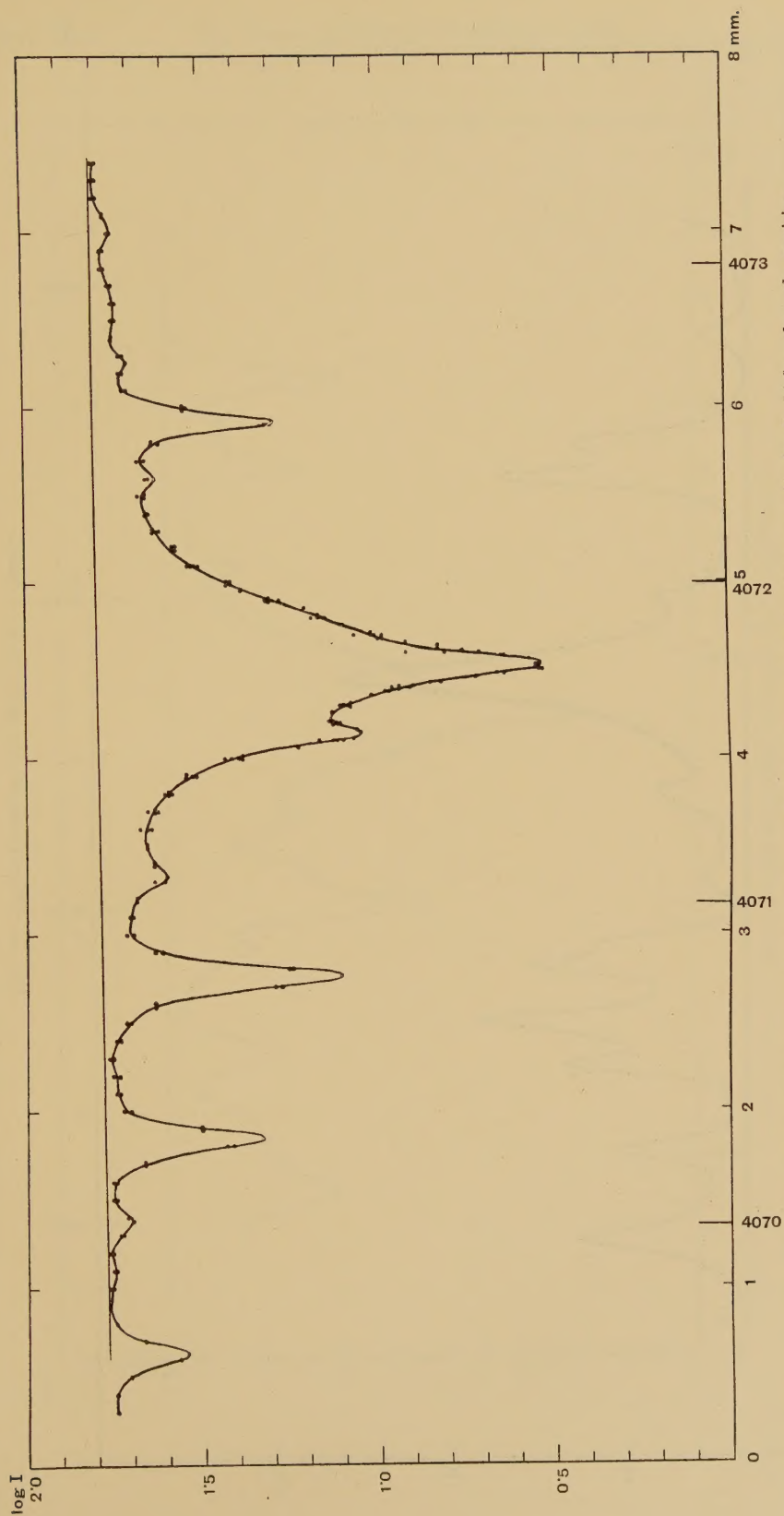


FIG. 4.—Observed profile of $\lambda 4072$, FeI; 8 plates. The background here is about 0.03 (in $\log I$) higher than the position used when computing the true central intensity.

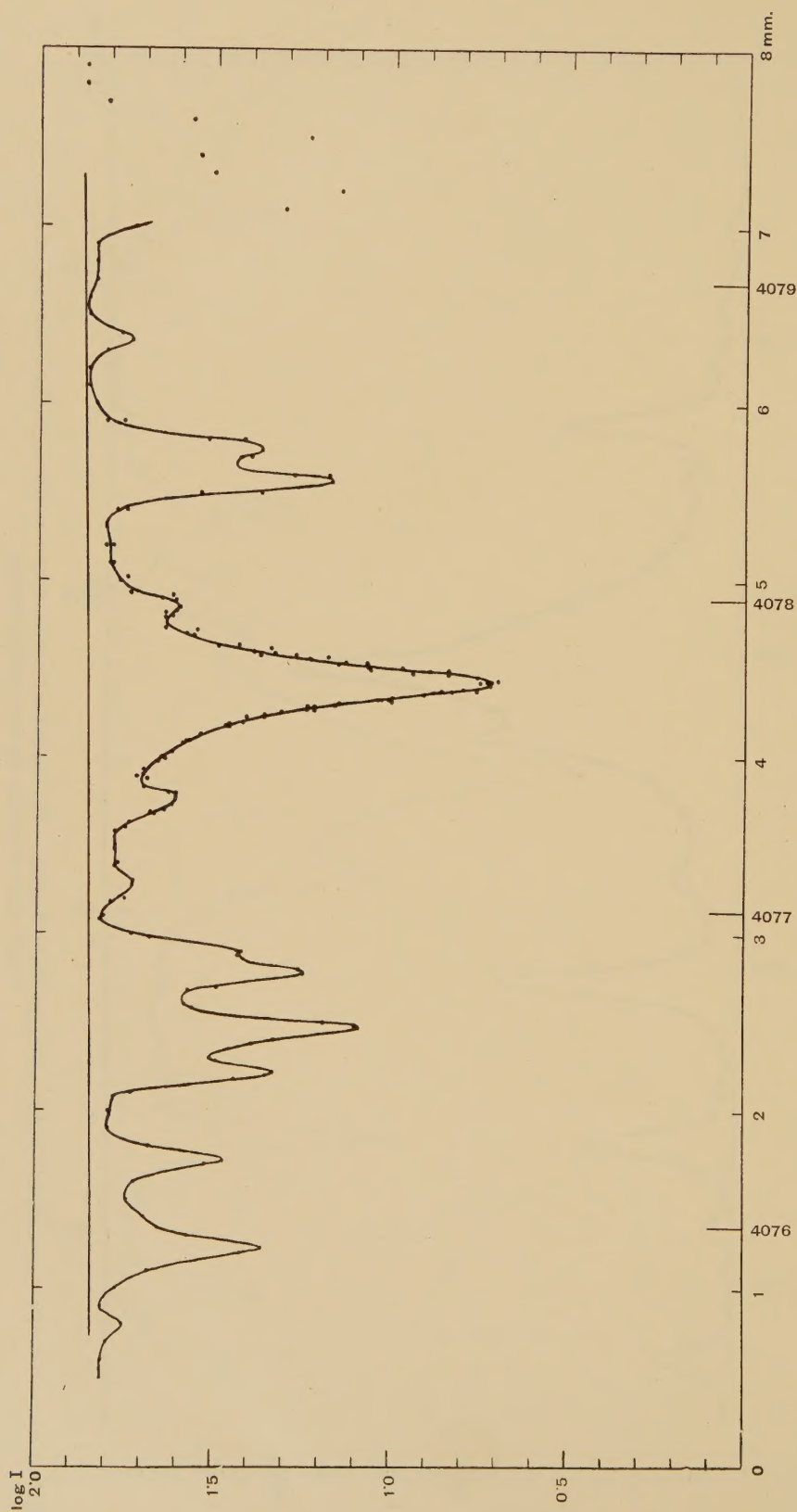
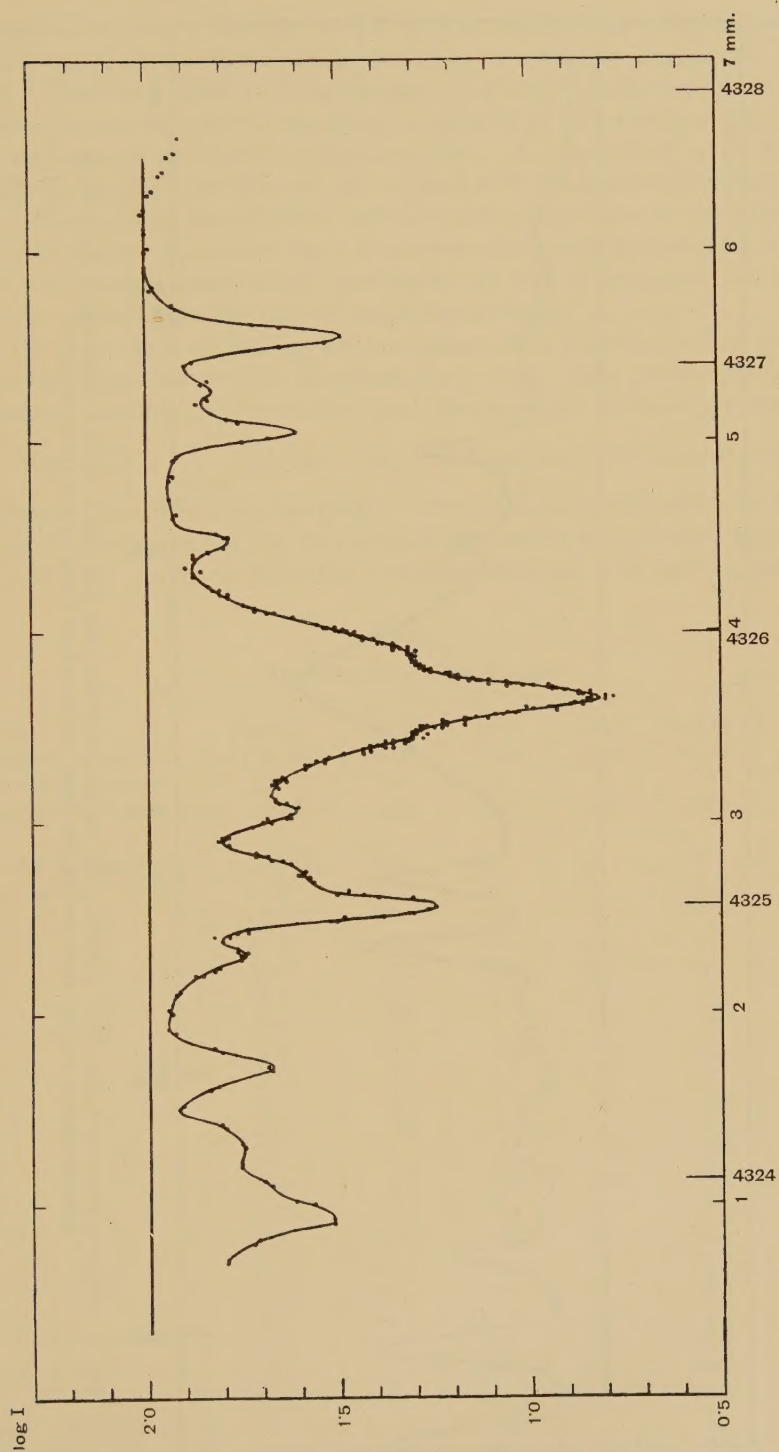


FIG. 5.—Observed profile of λ 4078, Sr II; 8 plates.

FIG. 6.—Observed profile of λ 4326, FeI; 16 plates.

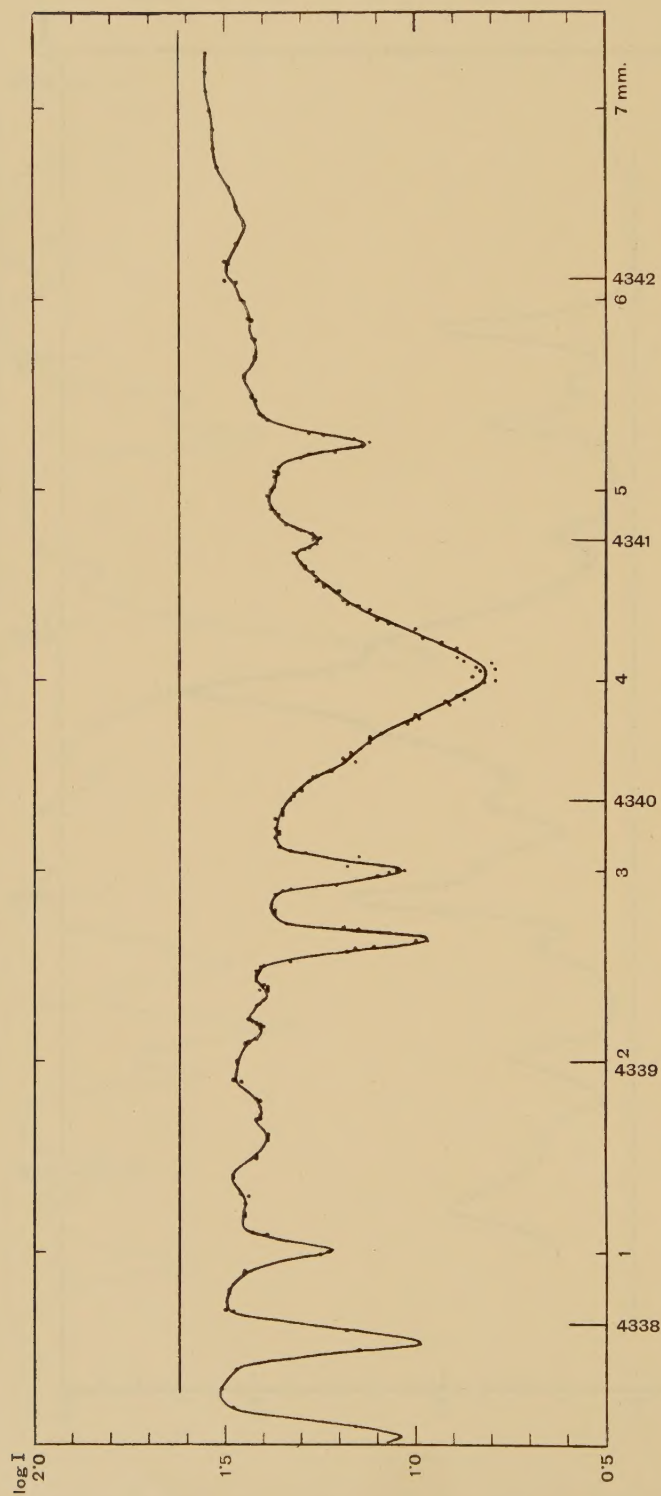


FIG. 7.—Observed profile of λ 4340, H γ ; 5 plates. The background at first sight may appear too high, but in reality this is not so. Microphotometer tracings show that the line wings extend at least 6 mm. (4.4 Å.) from the centre.

determination was discovered to be in error on account of scattered light ; again a small effect, but rather important in its consequences, as will be seen later. The third profile, which has been used for the reduction of the results of this paper, rests on the combined measures of 28 plates, ranging from a few minutes to 25 hours in exposure time. A comparison with the 1935 profile (I, 754) reveals differences in accord with the better definition noticed by ordinary visual inspection of spectrograms taken after the 30° prism had been refigured ; the curve has a somewhat sharper peak than before and is now very nearly symmetrical, practically all the improvement in the last respect occurring on the side of longer wave-length.

The procedure of finding the true shape of a Fraunhofer line from the observed shape was briefly described in I, 753. One guesses the answer, assumes a profile $L(x)$ for the line and distorts it by evaluating numerically the integral $O(x) = \int_{-\infty}^{+\infty} L(\xi)I(x-\xi)d\xi$, $I(x)$ being the instrumental profile and x distance measured along the plate. $O(x)$ should coincide with the observed line and it is easy, if tedious, to run through a series of successive approximations to $L(x)$, using as guide the residuals obtained between the calculated

TABLE I
Deduced True Line Profiles

Line	4046A.	4064	4072	4078	4326	4340
Scale of spectrum	0.53 A./mm.	0.56	0.55	0.56	0.70	0.73
Rowland number	30	20	15	8	8	20 N.
Equivalent width	1.08 A.	0.81	0.61	0.38	0.67	2.0
Profile (at intervals 0.025 mm.)	14.9 %	20.0	22	42	22	26
	13.9	17.7	22	39	12.2	23
	12.7	16.1	19.6	36	17.5	22
	11.5	14.2	17.6	31	23	21
	10.0	12.5	15.5	26	19.5	19.7
	8.1	10.5	13.0	19.4	15.5	18.2
	6.4	8.8	10.2	13.7	11.6	17.1
	4.7	7.0	7.6	8.2	8.3	16.1
	3.0	5.2	5.5	5.5	5.6	15.2
	2.2	3.2	3.4	4.1	3.5	14.8
	2.1	1.9	2.2	3.3	2.5	14.5
	3.0	2.7	3.6	2.6	4.2	14.7
	5.0	5.3	5.8	2.0	7.1	15.0
	7.0	7.7	8.3	10.2	10.4	15.6
	8.7	10.4	10.8	16.3	14.0	16.5
	10.4	12.2	13.3	22	17.2	17.7
	11.8	14.3	15.2	27	19.9	19.0
	13.1	16.2	17.1	33	21	20.3
	14.3	17.7	19.0	38	20.5	22
	15.3	19.6	21	43	19.0	24
	16.3	22	22	49	17.4	28
Mean residual in distortion correc- tion	0.1	0.1	0.1	0.1	0.3	0.2

$O(x)$ and the observed line. Generally the numerical integration was performed at intervals 0.025 mm. on the plate and the true profile computed for a distance not less than 0.25 mm. on either side of the line centre (scale of spectra, 0.5–0.7 Å./mm.). Outside this range, the effect of instrumental distortion is very small. For the unobserved outer part of the instrumental profile, extrapolation has been made, as before, on the assumption that the intensity continues to fall off as the square of the distance from the centre, and the Fraunhofer line intensity with which this is multiplied in the integration has been taken to have the average value 80 per cent., an empirical allowance for the wings of the line and the presence of other absorption lines. The extrapolated portion adds 0.4 per cent. to the central intensity in the computed $O(x)$, the deduced true line centre being lowered by about the same amount. The average residuals between the final computed $O(x)$ and the observed line are in Table I, the values being within the uncertainty of the observed profile. The same table also gives the computed profiles, including the deduced true central intensity of the six lines observed. All the lines have been observed by Allen at Canberra,* and it is of interest to compare the results (Table II):

TABLE II
Comparison of True Central Intensities

	Canberra	Cambridge
	Per cent.	Per cent.
λ 4046	3	2.1
4064	3	1.9
4072	3	2.2
4078	4	2.0
4326	5	2.5
4340	15	14.5

The method pursued by Allen differs from that described here in many details, and the agreement may consequently be regarded as not unsatisfactory. It is worth recalling that Allen's were the first extensive measures to appear with proper correction for instrumental distortion, vacuum iron arc lines being used as standards.

For the sake of completeness the total intensities of these six lines have been measured and are to be found in Table I. The values given are systematically smaller than Allen's (about 10 per cent. for the iron lines) and there is a large accidental scatter as well. It is clear, from the agreement of the central intensities, that these differences are not due to purely photometric errors. Although by no means impossible, there is as yet no definite evidence that the integrated intensities really vary, so that we suppose, as is very likely, that the discrepancies arise on account of the great difficulty of judging where the continuous background should be placed and how the blended lines should be smoothed over. With regard to blends all these lines are badly situated, and one cannot expect any great precision in the total intensity values. Central intensities are fortunately little affected by these particular troubles.

* *Memoirs of the Commonwealth Solar Observatory*, 5, 1934.

Errors

The above brief description of this observational work will by itself convey an illusory impression of simplicity, unless an account is also given of the likely sources of error and the attempts made to reduce or eliminate them. The errors may be divided into two classes : (A) those present in the observed profile, (B) those introduced in the process of correction for instrumental distortion.

(A) The errors of this group may themselves be subdivided as follows :—

- (a) Purely accidental errors.
- (b) Wrong plate calibration procedure.
- (c) Errors in transmission of wedge on calibrating spectrograph.
- (d) Errors from scattered light.
- (e) Errors from Eberhard effect.

(a) Ilford Process plates have a slow emulsion which with careful handling is very clean and has a very fine grain. Measures were made only over the most favourable range of density, and in these circumstances the results from plate to plate are very consistent and in excellent agreement. The deviation of one observed point from the final mean profile, averaged for all six lines measured, was 0.013 in log I. The average deviation of one reading at the six line minima themselves was 0.02 in the same scale. Since the number of separate readings for the line centre was three or four, except for $\lambda 4326$, where it was eight, it can be seen that the purely accidental error in the central intensity should be quite small (< 0.2 per cent. for all lines save $H\gamma$, where it should not exceed 0.5 per cent.) and is much less to be feared than the many sources of systematic error.

(b) Two additional virtues of Ilford Process plates, probably shared by all very slow, undyed photographic emulsions, are that the changes of shape of the characteristic curve with wave-length and with exposure time are very slow indeed. Tests made by the writer some time ago failed to show any detectible difference in this curve between $\lambda\lambda 4065$ and 4340 , or between exposure times 30 seconds and 15 minutes, in the density range used for all the work being described here.* The solar spectra were exposed for times varying between 15 seconds and 4 minutes and are at various wave-lengths $\lambda\lambda 4046$ – 4340 . The calibration exposures were for times 100–120 seconds and were always measured only at $\lambda 4227$. The behaviour of the emulsion is such that this variation in exposure time and in wave-length is of entirely negligible importance.

The interval between exposures and development was always kept long relative to the interval between solar and calibration exposures, in order that any possible changes of latent image with time should have no perceptible effect.

(c) It was shown in the earlier paper that errors in the adopted wedge transmission should not give rise to an error of central line intensity greater than a small fraction of 1 per cent. The only question that arises is whether

* *Observatory*, 58, 127, 1935.

the wedge has in any way altered since it was calibrated nearly two years ago. Four check plates were taken, using a diaphragm method, each giving five exposures of known intensity ratio for comparison with the wedge image.

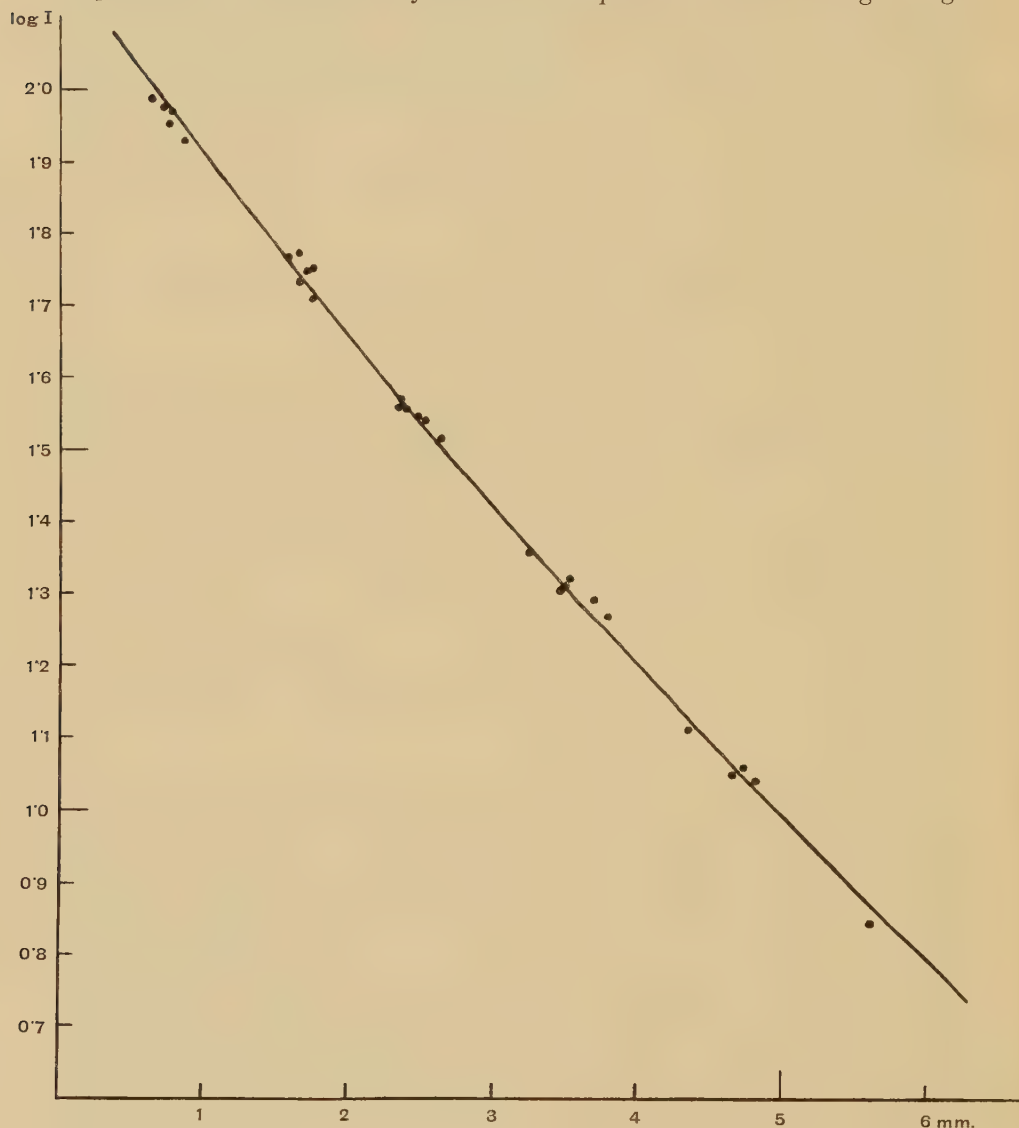


FIG. 8.—Wedge calibration check.

One plate was rejected, as it was taken on old stock, which showed very black edges on development and which proved to have been left over from an eclipse expedition. The other three showed excellent agreement with the original calibration curve (fig. 8), which seems not to have changed in the least.

(d) Light scattered inside the spectrograph is negligible (I, 747). Not only did the monochromator limit the range of spectrum admitted to 30 or

40 Å., but a shorter slit was used than in the λ 4227 work, which would cut down still further the amount of light in the instrument. Light scattered by the photographic emulsion is quite unimportant, on two counts. Contact prints of a spectrograph slit, made on Process plates in monochromatic light of wave-length λ 4358 Å., showed that the spread of the image due to turbidity of the emulsion is extremely small in normal circumstances, while in any case the scatter will also affect the krypton line measures and will amount to a very slight diminution of resolving power, fully corrected in the reductions described already.

(e) One of the chief bugbears of workers in spectrophotometry is Eberhard effect, mostly on account of the fact that it is difficult to detect and varies greatly, in a way still imperfectly understood, with the different conditions of plate emulsion, development, etc., which may be encountered. All the plates used in this investigation were brush-developed in M-Q at about 18° C., in a developing tank with thermostat. To avoid Eberhard effect it has been recommended in technical photographic literature that plates should be quite fully developed. Our experience at the Solar Physics Observatory does not altogether support this, as the trouble generally begins to be noticeable only in the presence of developer fog. It is possible that the particular range of density utilized by the microphotometer may have some influence in the matter. Be that as it may, over-development here was studiously avoided and the procedure seems to have been successful, as the following special tests showed.

An enlarged image of the strip filament of a tungsten lamp was projected on and perpendicular to the first (vertical) slit of the spectroheliograph of the Observatory. The magnification was about eight diameters, so that for distances of the order of 1 cm. from the centre, the filament image could be taken to be of uniform brightness. In the particular arrangement of the spectroheliograph which was employed, the optical train consists of a simple collimator lens, two plane mirrors, a plane grating and a simple camera lens. The first slit was used very wide, the second slit had a width 0.04 mm. and was set to transmit light near λ 4300 Å. The lens projecting the filament image was diaphragmed with a simple sector shutter, which turned around the optical axis, and by rotating this steadily by hand, timing the motion with the help of a graduated circle and a clock ticking seconds, a uniform diminution of light intensity could be obtained, down to 6.6 per cent. of maximum light and back again. Now, keeping the filament image and the plate fixed, the whole spectrograph could be moved horizontally with a uniform slow motion and, by simultaneously keeping the lamp current constant and rotating the shutter, an artificial "absorption line" of arbitrary dimensions could be built up on the plate. In practice the shape aimed at was very near that observed for λ 4326. The illumination on the grating fell always well within the ruled area and so the transmission of the whole spectrograph could be regarded as constant during the experiment. Since the whole run, recording a "line" and "continuous spectrum" on each side, covered only about 6 mm., variations of intrinsic brightness along the filament image could be neglected. The graph of light fed into the spectrograph plotted against

time had a short stretch of constant minimum illumination, but this is rounded off in the representation built up on the plate, on account of the finite width of the second slit and of the penumbra of the shadow of the slit jaws,

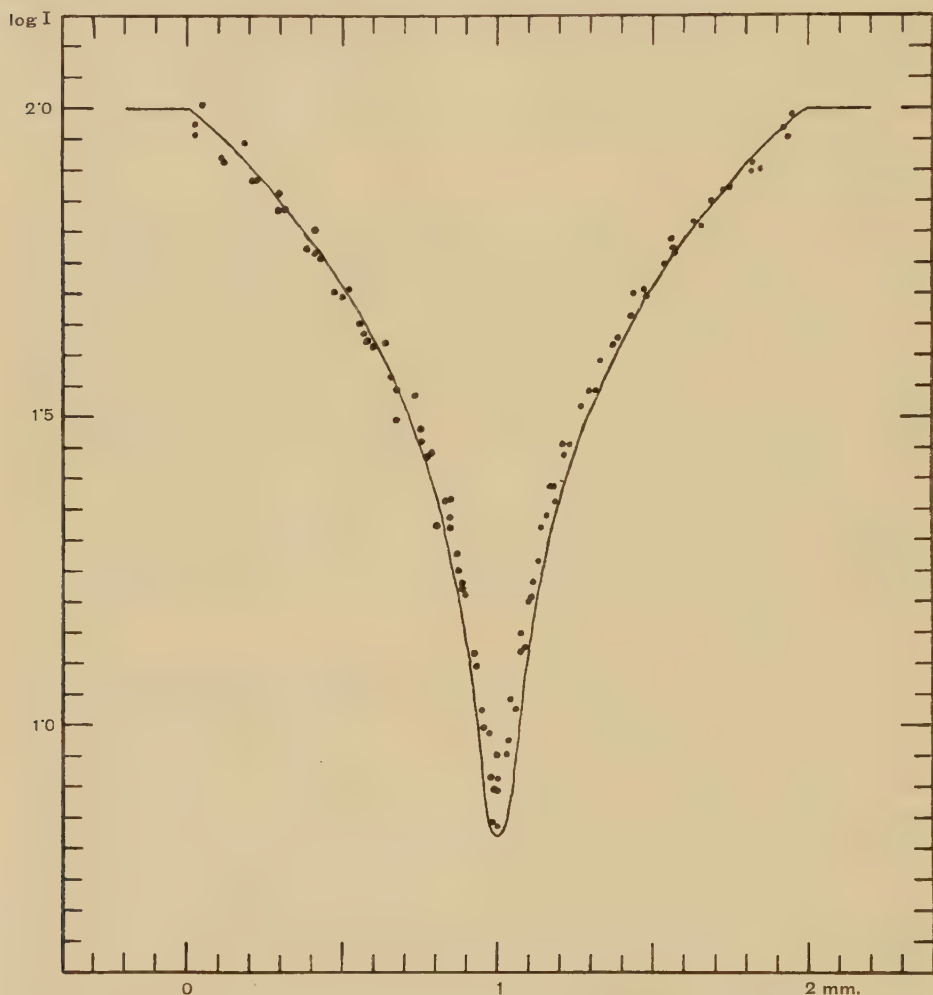


FIG. 9.—Artificial absorption line for testing for Eberhard effect; observed points compared with the true light distribution.

as well as small diffraction effects. It was estimated that the intensity effective on the plate would barely fall to the mentioned 6.6 per cent.

The plates used were Imperial Process New Series, which for some years have been made by the Ilford Company and are stated by them to have an emulsion identical with that of Ilford Process plates. Calibration and development were carried out exactly as for the solar spectrum. Eight plates were measured, and in fig. 9 the observations are compared with the true light distribution. Since the motion of the spectroheliograph could not be kept perfectly constant from run to run, it was impossible to keep the "line width" completely invariable, so that this had to be measured on each plate and

corrections made to reduce all to the same value. It was noticed that in the final plot the scatter of individual points is appreciably greater than is normal in solar work. The observed central intensity is 8 per cent.; the excess above the true value diminishing steadily from 1.4 per cent. at the centre of the "line" outwards.

In view of the fact that the conditions of illumination in this experiment are not quite the same as in normal spectrophotometry and that there may be a small amount of light scattered from the jaws of the second slit, these results must be regarded as entirely satisfactory. The deviation from the true central value is in the *opposite* direction from that which would be caused by Eberhard effect. We take it that the latter has no appreciable influence on our work.

(B) Errors arising in the correction for instrumental distortion must also be discussed. The source of the krypton spectrum was an ordinary Geissler tube, filled at a pressure of about 3 mm. mercury with spectroscopically pure krypton containing a trace of xenon. It was used at ordinary room temperatures and was excited by a small transformer run from the mains (frequency 50 cycles/second). The lines used were $\lambda\lambda$ 4274, 4319 and 4320 Å. With very long exposures there was slight overlapping of these last two lines, but the necessary small corrections can be found and applied without difficulty.

Potential sources of error are :

- (a) Incorrect photometric procedure.
- (b) Changes during long exposures.
- (c) Scattered light in the spectrograph.
- (d) Light scattered in the photographic emulsion.
- (e) Erroneous extrapolation of instrumental profile beyond the observed portion.
- (f) Structure in the lines themselves.

Some of these are of a nature similar to those already discussed, but as their mode of action here is somewhat different they must be considered fully once more.

(a) The earlier plates used for deriving the instrumental profile were calibrated with the wedge, but then doubts arose as to the wisdom of using a steady source, viz. an ordinary gas-filled tungsten lamp, to calibrate plates exposed to the flickering krypton tube. Hence a great many of the later exposures were calibrated with a tube sensitometer,* using another identical krypton tube for illumination, with a smooth magnesia block for diffuser. In this case, spectrograph and calibration exposures were run simultaneously, for times generally identical but occasionally differing by a factor of not more than two. Possible errors on account of intermittency or of differences of exposure time could in this way be definitely eliminated. The krypton tube for the sensitometer was used without a filter, and since the plates were not colour sensitive they were exposed effectively to radiation in the region $\lambda\lambda$ 4300–4500 Å., which contains all the strong krypton lines in

* For tests of the sensitometer see *M.N.*, 96, 593, 1936.

the blue-violet. For the purpose of determining the characteristic curve, this region is sufficiently near the wave-length of the lines measured. The results with wedge calibration and with sensitometer calibration agreed well, the latter perhaps showing a tendency to give a slightly sharper profile. The mean instrumental profile includes both series of plates.

(b) One of the weaknesses of this method of assessing the optical performance of the spectrograph is that it requires very long exposures, a necessity which makes it more likely that changes will take place, shifting or blurring the spectral lines and giving an instrumental profile which is too wide. Changes most to be feared are in temperature and in atmospheric pressure. Fortunately the thermostat controlling the temperature of the McClean spectrograph has proved very reliable, and once it has started and has been given time to settle down to its work the observer can be confident that the temperature of the prisms will stay constant to $0^{\circ}\cdot 1$ C., provided the temperature of the surrounding room does not fluctuate too violently and does not rise above the level within the instrument. Two mercury thermometers are provided in the spectrograph and were watched during exposures as a matter of routine. Under these conditions there is no trouble with temperature changes.

Variations in atmospheric pressure are more serious, since there is no possibility, with the instrument in its present form, of preventing these giving rise to quite considerable movements in the spectrum lines. It has been realized for many years that the change of refractive index of air consequent upon an alteration in atmospheric pressure will alter the deviation of a prism spectrograph, unless steps are taken to counteract the effect.* In the wave-length region containing the krypton lines used, 10 mm. change in barometric height produces with the McClean spectrograph a shift of about $0\cdot 11$ Å. It is by no means impossible to get such a pressure change in twelve hours in this country, although conditions are fortunately usually steadier than this. The average change of barometric height per hour during six of the earlier exposures, which happened to be made in less settled weather than most of the later photographs, was about 0.3 mm. Now the centre part of the instrumental profile can be, and has been, measured on exposures of about five minutes duration, in which time normal barometer shifts would be quite negligible, even for refined wave-length measures. Only short exposures were used for the profile centre. On exposures of one hour the measured region was between about $0\cdot 07$ and $0\cdot 10$ Å. from the centre, whereas the corresponding barometer shift would normally not exceed $0\cdot 003$ Å. On exposures of twenty-four hours the measured region is around $0\cdot 2$ to $0\cdot 65$ Å. from the centre, while the normal barometer shift may be about $0\cdot 08$ Å., although in some cases it was actually less than one-tenth of this. Inspection of the slope of the observed profile shows that pressure changes must have had negligible effect on it, a conclusion borne out by results from plates which were taken at times when the barometer happened to be unusually steady.

* Merton, *Proc. Roy. Soc., A*, **113**, 704, 1926; Elliott and Cameron, *Journal of Scientific Instruments*, **14**, 28, 1937.

It is worth while adding that the spectrograph is supported on brick pillars and that tests have shown it to be immune to normal vibration such as may be caused by persons walking about in the same room.

(c) It was deemed unnecessary to use a monochromator with the krypton tube, since the spectrum lines measured contain a far higher proportion of the total light entering the spectrograph than is the case in the solar spectrum. This optimism received a severe check when it was discovered, by a fortunate accident, that the greater part of the light being observed in the extreme outer parts of the line profile, which are of course very faint relative to the centre, was due to scatter in the spectrograph. This was found to be a consequence of the use of a needlessly long slit and of a condensing lens of needlessly big aperture, which threw a good deal of light on the walls of the instrument. Cutting down the length of slit by a factor 15 and the area of condensing lens in a still greater proportion, the observed intensity in the outermost parts of the profile was reduced by a factor of about 4 and what is now left is certainly not due in any appreciable degree to scattered light. Much additional time and labour had to be spent in redetermining the outer parts of the curve as a result of this trouble. The effect on the centres of the Fraunhofer lines is considerable; with the erroneous line profile the solar observations were being over-corrected and the deduced central intensities were generally almost exactly zero, whereas with the revised corrections the same observations give central intensities about 2 per cent.

(d) The effect of light scattered in the photographic emulsion is normally included in the instrumental profile and thus correction for it is automatically made, but since the Process plates used for the solar exposures and for the central parts of the instrumental profile had to be replaced by a faster emulsion (Ilford Zenith 650) for the outer parts of the instrumental profile, this statement has to be qualified. If the image spread, as a result of turbidity of the emulsion, is much greater with the Zenith plates, it is conceivable that the instrumental profile might be affected. However, this does not seem to be the case. As already mentioned, contact prints of a spectrograph slit, of width about 0.05 mm., were made in monochromatic light $\lambda 4358 \text{ \AA}$. on a number of plate emulsions. Apart from streaks due to reflections from irregularities on the jaws of the slit, the images were sharper than any spectrum line the writer remembers having seen produced by any spectrograph. They were of a different order of sharpness altogether from the best line images in the McClean spectrograms, and it seems certain that scatter in the photographic emulsion must play a quite subordinate rôle in these matters. This might also be expected in view of the resolving power of a photographic plate, which is about 100 lines per mm. for a Process emulsion and about 30 per mm. for a fast coarse-grained emulsion.*

(e) The instrumental profile was observed down to an intensity 0.00027 of that at the centre. It was scarcely practicable to push the measures to

* It is not always realized by observers that a photographic optical instrument working at a relative aperture greater than $f/20$ can never attain the maximum performance indicated by the wave theory of light, on account of the lack of resolving power of photographic emulsions.

still fainter regions, so that resort was made to extrapolation, as explained above. The effect of that part of the correction which relies on extrapolation is roughly to reduce the computed true central intensity by 0.4 per cent. Apart from the uncertainty of the extrapolation (which should be quite small), there is a possibility of error in the approximate method of computation, but this has been tested for λ 4326 and seems unlikely to lead to an error greater than 0.1 per cent. The total probable error from both causes may fairly be estimated as 0.2 per cent.

(f) Finally, there is the question of the true structure of the krypton lines themselves. It has been assumed all along that the true line width is negligible in comparison with the width of the narrowest profile the McClean spectrograph is capable of giving. As far as observations exist, the theoretical prediction of the line width, based on the kinetic theory of gases and Doppler's principle, is fulfilled. According to a recent publication of C. V. Jackson,* the particular lines employed in this work have a "width" 0.008 Å. and show no certain signs of fine structure, although they have been examined with Fabry-Perot étalons by Humphreys, by Kopfermann and Wieth-Knudsen, and by Jackson, while the latter has also failed to resolve them with a reflection échelon of resolving power greater than 600,000. There is a possibility that each line has weak satellites due to isotopes, but the échelon observations show that these must be very faint and they would also be expected to be comparatively near the main line, relative to the scale of the instrumental profile we are investigating.

There exists no technique at present by which the outer parts of the true krypton profile can be measured, since an interferometer of some kind would be needed and Jackson states that the background produced by such apparatus is "considerably greater than 1 per cent." †

The krypton spectrum is not very sensitive to pressure broadening, the particular lines under consideration showing an increase of width about 20 per cent. at 10 mm. mercury (Jackson), which is three times the pressure at which our discharge tubes were filled. It may be remarked that these two tubes give identical results, and with neither has any continuous spectrum been photographed.

The "width" referred to in Jackson's interferometer work is presumably that where the intensity has one-half its maximum value, at which point the observed instrumental profile is about seven times as wide as the krypton line. Strictly speaking, the true instrumental profile is just a little narrower than that observed, but the difference between the two is so very small that it can be quite safely neglected.

If, notwithstanding evidence at present available, the krypton lines should possess faint wings, comparable in intensity with those shown in the instrumental profile, the corrections to the Fraunhofer lines are too great and we have deduced central intensities which are too low. However, those parts of the krypton lines which have been measured by interferometers are in such good agreement with expectation and are so much narrower than the peak of

* *Phil. Trans.*, **236**, 1, 1936.

† Private letter.

the instrumental profile that it seems quite justifiable to assume that they continue to obey theoretical prediction and are equally satisfactory in regions further removed from the line centre.

To make a precise estimate of the probable error of the final central intensities is impossible. Internal agreement of a series of observations, made under similar conditions with the same instrument and by the same observer, is notoriously an unreliable guide to the real errors of the results, particularly when the aim is to attain the greatest precision a particular method can give. There is no question that the results of the present paper are more reliable than the earlier measures of $\lambda 4227$, but, on the other hand, additional experience has emphasized the difficulty of eliminating all sources of systematic error with complete certainty, so that it is perhaps wise to continue to use the same probable error as before, viz. 0.7 per cent. of the adjacent continuous spectrum. Judged solely on the basis of internal consistency, the accuracy is appreciably better than this.

Discussion

There are four points in the results which call for comment.

(1) The comparison given above (Table II) between our results and those of Allen shows differences about what would be expected on the basis of the respective probable errors estimated by the observers themselves. It seems that at last we are really able to obtain the central intensities, at least of the strong Fraunhofer lines, free of serious systematic error and with a probable error 1 per cent. or less of the continuous spectrum.

(2) It now seems likely that the majority of strong Fraunhofer lines of metallic origin (Rowland number greater than 8) have central intensities as low as 2 or 2.5 per cent., whereas the hydrogen Balmer lines, $H\alpha$, $H\beta$, and $H\gamma$, show values around 15 per cent., according to consistent observations of several independent observers, *e.g.* Minnaert, Thackeray and Allen. The hydrogen lines are also remarkable for their very extensive wings, which are much broader than is usual with metallic lines. There seems every reason to suppose that these characteristics are to be associated with Stark effect.

(3) During the reductions it was noticed that $\lambda 4078$, $SrII$, after correction for instrumental distortion, is asymmetrical at its centre, the point of lowest intensity being roughly 0.03 Å. to the red of the centre of gravity of the whole line. The effect is small and would probably have been attributed to some measurement error had not Dr. Thackeray informed the writer that he had observed very much the same asymmetry himself while working at Mt. Wilson. It is possible that the cause is blending with adjacent faint lines at $\lambda\lambda 4077.582$ and 4077.836 . There are slight signs of a similar effect with other lines, but as far as present observations go it is with them even more difficult to make sure that faint blends are not the cause. However, asymmetry in the outer parts of several Fraunhofer lines, including $\lambda 4078$, has

been found by Unsöld,* by d'Azambuja,† and by H. H. Plaskett.‡ Thackeray has observed asymmetry in the cores of a number of lines. In all cases the effect is in the same direction, the side of longer wave-length being the steeper, and there is thus good reason for supposing it to be real and not due to blends. The evidence concerning variation with respect to position on the disc is at present conflicting and requires further investigation.

(4) The close agreement between the central intensities of the five metallic lines measured here is quite remarkable, all five lying within a range of 0.6 per cent., which also covers the central intensity of λ 4227. All the lines are of the same order of width and total intensity, and it is natural to suppose that the same common cause is at work in determining the central intensity. Several mechanisms have been suggested from time to time to explain why lines are not perfectly black at the centre. Of these, fluorescence and "inter-locking" appear to be ruled out. There is no distinction in behaviour at the centre between the resonance lines λ 4078, *SrII*, and λ 4227, *CaI*, and the non-resonance lines λ 4046, 4064, 4072, *FeI*, and since the first two should be relatively immune to these effects, we may suppose that the other three probably are so too. We leave out of account λ 4326, *FeI*, since it is a close blend of two lines, of which one, probably much the weaker, is a principal line ($a^5D_4 - z^7F_3$).

A simpler and more satisfactory explanation for the observed central intensities is Doppler effect. Velocity in the line of sight may be provided by (1) the separate motions of individual atoms, or by (2) the movement of matter in bulk. For a given average velocity in the line of sight, these do not necessarily have an equal effect, the latter under some conditions being able to produce a greater distortion of the structure at the line centre. An example of (1) is provided by the thermal motions of atoms, which will always be present, even in an otherwise static atmosphere, but at the temperature of the reversing layer, say 6000° C., the velocities are only 1 to 2 km./sec. for substances such as calcium, strontium and iron. The effect of such velocities on lines of Rowland number 8 and upwards is only perceptible at the line centre, where an otherwise perfectly black line could be given an intensity about 0.5 per cent., a result not very sensitive to the particular atmosphere model with which the computation is made. On the other hand, it is conceivable that in certain circumstances a motion of matter in bulk, with the same average line of sight velocity, would be quite sufficient to fill in the line centres to the 2 or 2.5 per cent. which is observed. Owing to the finite area of slit and to imperfect seeing, the least area of the solar disc contributing light to the measured part of any one spectrogram in this work must be of the order of 10^7 km.². Suppose we picture such an area of the Sun's surface as made up of independent units having the following properties: (a) each covers a disc area not exceeding 10^5 km.², (2) each has its own peculiar line of sight motion, the average velocity for all areas being 1 to 1.5 km./sec., (c) each

* *Z.f.P.*, 44, 793, 1927.

† *Annales de l'Observatoire de Paris (Meudon)*, VIII/2, 66, 1930.

‡ *M.N.*, 91, 870, 1931.

produces a perfectly black absorption line ; then the line measured under the present observing conditions would have the shape actually observed, the central intensity being 2 to 2.5 per cent. In condition (c) we are supposing that each small area is sufficient to cover a column of gas which itself is large enough for the current theories of line formation in a static atmosphere to apply. This is plausible when we recollect that we are concerned only with the centres of very black lines, for which the atmosphere is very opaque, and for which the column of atoms doing most of the work of line formation must be relatively short. (The atmospheric levels mostly concerned are probably in what is usually regarded as low chromosphere.)

The idea of turbulent motion in the Sun's atmosphere is a very old one. For a long time observers have known that even the least disturbed portions of the disc show a rapidly changing granular structure, with granules having diameters of the order of 1000 km. and mean life times of only two or three minutes. But apart from specific eruptions, sun-spots, prominences, etc., not very much is known of the velocities which may be involved. *Chromospheric* velocities have been investigated by Unsöld,* who finds 15 km./sec. for calcium atoms. Menzel,† in his work on the flash spectrum, mentions 12 km./sec. in lower layers to 20 km./sec. in higher layers, again in the chromosphere, although it seems unlikely that his spectrograms have more than just enough resolution to measure such quantities. (These chromospheric velocities are measured *parallel* to the solar surface.) Evershed‡ has measured local wave-length shifts on the disc of about 0.2 km./sec. These, however, "appear mostly at a distance from the centre of the disc, and rarely at the centre", and he suggests that, if Doppler shifts, they must be due mostly to velocities parallel to the solar surface. It is to be noted that these observations are likely to refer more nearly to the atmospheric levels we are considering than do chromospheric observations, but, on the other hand, the local velocity irregularities they give will generally be too small, on account of the smoothing effect of bad seeing. On the whole the present position is that there is no evidence against a vertical turbulent motion of the order of 1 km./sec. in the upper parts of the reversing layer. Our general knowledge of the atmosphere leads us to suppose such motions not at all unlikely, but there is very little positive evidence for them apart from the central intensities we have been measuring.

Addendum

Since the above was written, Dr. Allen's latest paper § has arrived, and it appears that my work offers substantial support to his conclusions. First, his recent observations at Mt. Wilson, using terrestrial atmosphere lines to provide the instrumental distortion correction, have led him to believe that his Canberra central intensities were under-corrected, which agrees with the results of this paper. Secondly, he finds it necessary to postulate turbulence in order to explain the behaviour of relatively faint lines in the red region of the spectrum. My "movement of matter in bulk" and "separate motions of

* *Ap. J.*, **69**, 209, 1929.

† *M.N.*, **94**, 96, 1933.

‡ *Lick Observatory Publications*, **17**, 296, 1931.

§ *Ap. J.*, **85**, 165, 1937.

individual atoms" are in effect alternative ways of describing his "large scale turbulence" and "small scale turbulence" respectively. He comes to the conclusion that his observations with regard to both line profiles and curves of growth are satisfied equally well by a "large scale turbulence" "with a Maxwellian velocity distribution and a most probable velocity of 1.8 km./sec." or a "small scale turbulence" with velocity 1.5 km./sec. As has been indicated, my observations are in agreement with the former alternative, but with a slightly lower average line of sight velocity—1 to 1.5 km./sec. being sufficient. The agreement, however, is quite close, even without allowing for the fact that Allen's velocities must refer to atoms lying in the reversing layer at systematically lower levels than do mine.

THE PHYSICAL STATE OF JUPITER'S ATMOSPHERE.

Bertrand M. Peek.

In two papers, published in 1923 and 1924, H. Jeffreys * investigated the physical constitution of the outer planets. His conclusion was that they were cold, and this was subsequently supported by observations, which indicated a temperature of about 150° A. for the surface of Jupiter. At the end of the second paper he suggested on dynamical grounds, based upon the observed value of the ellipticity, that Jupiter was composed of a rocky core, surrounded by a thick layer of ice, which was covered by an atmosphere whose depth was about 9 per cent. of the radius of the planet, or more than 6000 km.

The present paper was originally conceived with the idea of proving that such an atmosphere cannot exist; for either the temperature must rise with the depth at a rate that will certainly not admit of the interior being regarded as "cold," or the pressure, at quite surprisingly small depths, becomes so great that, even if the gases do not liquefy on account of their low critical temperatures, the density becomes equal to that of their liquid or solid states. After nearly the whole of the work had been completed, the author was informed by Dr. Jeffreys that R. Wildt,† in a paper published in 1934, had already shown that the theory of an extensive atmosphere must be abandoned on the grounds of the great densities involved. As, however, there is little similarity between Wildt's published work and the present method of approach to the subject, and as the results now obtained show even smaller maximum depths for the atmosphere than were indicated by Wildt, it has been decided to present the paper in practically its original form.

We proceed, then, to examine three models of the atmosphere, two being extreme cases and the third intermediate.

Of the two extreme cases the first is that of convective equilibrium in the absence of condensation, where the pressure, density and temperature

* *M.N.*, 83, 350, 1923; and 84, 534, 1924.

† *Veröffentlichungen der Universitäts-Sternwarte zu Göttingen*, Nr. 40, 1934.

obey the ordinary adiabatic laws; the second is the case of an isothermal atmosphere.

Except where otherwise stated, C.G.S. units are employed, the temperature being measured in degrees absolute.

We take as datum-level the stratum of the atmosphere that gives rise to the observed temperature of 150° .

The Adiabatic Model

If P , ρ , T be the pressure, density and temperature whose values at the datum-level are P_0 , ρ_0 , T_0 , we have the following well-known relations:—

$$P_0 = \frac{R}{\mu} \rho_0 T_0$$

$$= 1.24 \times 10^{10} \rho_0 \mu^{-1}, \quad (1)$$

$$\frac{\rho}{\rho_0} = \left(\frac{P}{P_0} \right)^{\frac{1}{\gamma}}, \quad (2)$$

$$\frac{T}{T_0} = \left(\frac{\rho}{\rho_0} \right)^{1 - \frac{1}{\gamma}}, \quad (3)$$

R being the universal gas constant, μ the molecular weight of the gas and γ the ratio of the specific heats.

Also the dry adiabatic lapse rate, Γ , is given by

$$\Gamma = \frac{g_J A}{c_p}$$

$$= \frac{g_J \mu}{R} \cdot \frac{\gamma - 1}{\gamma}$$

$$= 3.12 \times 10^{-5} \mu \left(1 - \frac{1}{\gamma} \right), \quad (4)$$

where A is the reciprocal of the mechanical equivalent of heat, c_p the specific heat at constant pressure and $g_J (= 2.65g)$ the value of gravity on Jupiter, which may be treated as constant over the range of depths here considered.

The above determine the adiabatic relations between pressure, density, temperature and depth below the datum-level, when the values of P_0 , μ and γ are given.

The Isothermal Model

Considering next the case where the temperature has a constant value of 150° at all depths, we have, as in (1),

$$P = 1.24 \times 10^{10} \rho \mu^{-1}, \quad (5)$$

and if z be the depth,

$$\frac{dP}{dz} = g_J \rho$$

$$= 2.10 \times 10^{-7} \mu P;$$

whence

$$dz = 4.8 \times 10^6 \mu^{-1} P^{-1} dP$$

and

$$z - z_0 = 4.8 \times 10^6 \mu^{-1} \log_e \frac{P}{P_0}, \quad (6)$$

z_0 having zero value for depths measured from the datum-level.

Before proceeding to the third and intermediate case, we shall obtain from the above numerical values indicating the variation of pressure, density and temperature with the depth. In order to accomplish this we must assign values to μ , γ and P_0 .

The first two depend only on the nature of the gases of which the atmosphere is constituted ; and, as we wish to derive a maximum value for the depth, we shall consider only gases with low molecular weights, in order that the pressure may be kept at a minimum. The spectroscope has shown the presence of considerable quantities of methane, CH_4 , and traces of ammonia, NH_3 , for which μ has the values of 16 and 17 respectively. These are the two lightest gases with the exception of hydrogen and helium. The two last might be present in large quantities without being revealed by the spectroscope, but it is difficult to conceive of the heavier CH_4 abounding in the upper strata, if the lower atmosphere were largely composed of H_2 or He . A large amount of nitrogen may be present, but its molecular weight of 28 would add considerably to the pressure. It may be noted that, if hydrogen and nitrogen were present in approximately equal volumes, their mean molecular weight would be about equal to that of methane ; if in equal weights, it would be about that of helium.

For the purpose of numerical evaluation in the adiabatic and isothermal models we treat the atmosphere as though it were composed entirely of methane.

For this gas

$$\mu = 16, \quad \gamma = 1.3.$$

There remains to be chosen an appropriate value of P_0 . Since, however, we desire to keep the pressure at a minimum, we shall be satisfied if we can assign to it a lower limit. And here, fortunately, we can again invoke the aid of observation ; for the spectroscope provides evidence that above the visible (or, more strictly, the photographic) surface there exists the equivalent of one mile of methane and ten metres of ammonia at normal temperature and pressure.* The pressure at Jupiter's surface due to these gases alone is easily calculated to be 0.335 atmosphere, which is the value we accordingly adopt for P_0 . The only assumption implied is that the stratum which is responsible for providing the light in the continuous spectrum is the same as that which emits the radiations from which the temperature is derived. As the latter are in the infra-red, it is reasonable to suppose that they emanate from a lower rather than a higher level than the photographic rays, and that the true value of P_0 may be higher but cannot be lower than that adopted.

* *Nature*, 135, 219, 1935.

In fig. 1 are shown the adiabatic relations between depth in kilometres, temperature and pressure in atmospheres, as calculated for methane. The figures written against the curve in brackets show the corresponding densities.

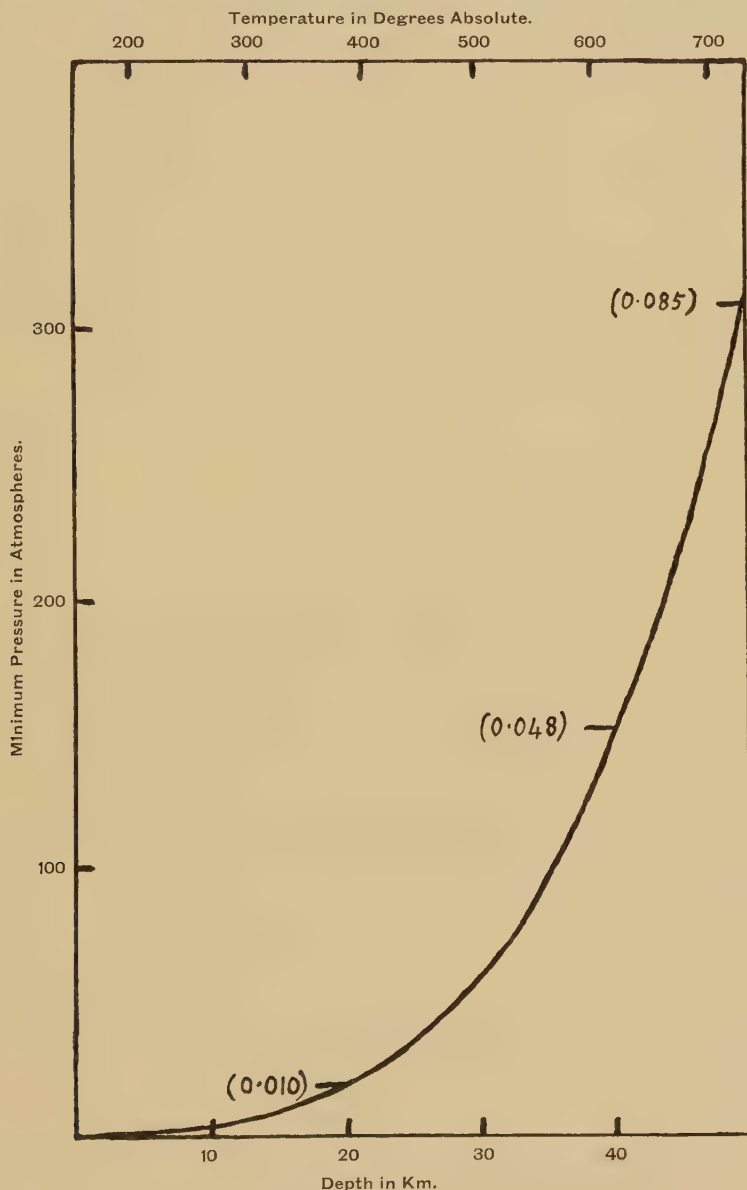


FIG. 1.—The Adiabatic Model.

It will be seen that, although the density does not at first rise unduly rapidly, and at 50 km. has only attained to that of solid hydrogen (0.08), the temperature, even at 20 km., is above that of boiling water. The lapse rate for an atmosphere composed entirely of hydrogen would be $1^{\circ}.82$ per km. instead

of $11^{\circ} \cdot 6$ for methane, so that in this extreme case the temperature of boiling water would be reached at a depth of 123 km. below the radiating surface.

Fig. 2 represents the isothermal relations ($T = 150^{\circ}$) between depth in kilometres, pressure in atmospheres and density, also for methane. We

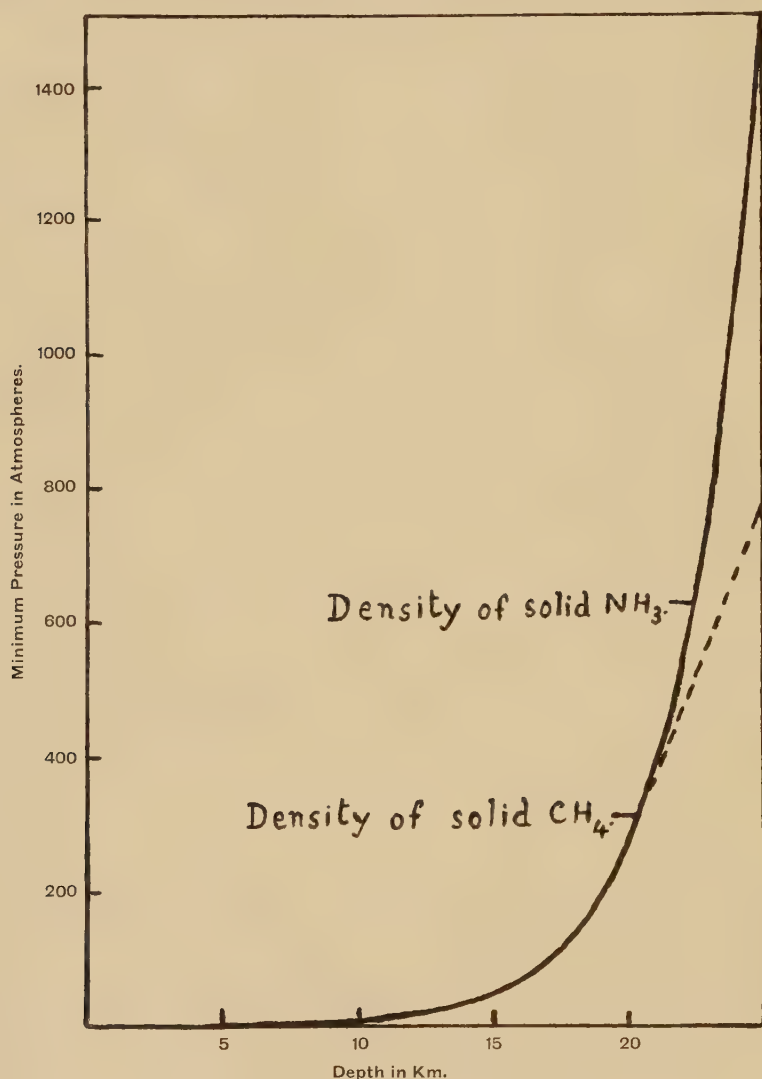


FIG. 2.—The Isothermal Model.

see that at a depth of just over 20 km. the density of solid methane (0.42) is attained, while, if the gas laws could still hold, we should have that of solid ammonia (0.82) at 22.5 km. ; actually, no doubt, the gas laws would become invalid before a depth of 20 km. was reached. The dotted line shows the manner in which the pressure would continue to increase, on the assumption that at the density of solid methane the gas would become incapable of further compression. In the extreme case of pure hydrogen

the density of the solid state would be found before the depth reached 150 km.

Wildt names 600 km. as the maximum depth that could be attained before an isothermal hydrogen atmosphere, presumably still obeying the gas laws, would become compressed to the mean density of the planet; but if the depth be calculated from (6), with a plausible assumption as to the value of P_0 , we find that this state occurs at a little less than 200 km.

Since the two models so far considered have led to very small limiting depths, the first on the grounds of temperature, the second because of the rapidity with which the density of the solid state is attained, we shall now examine an attempt to effect a compromise.

It is well known that in the Earth's atmosphere the mean lapse rate increases with the altitude, approaching the adiabatic rate most nearly just below the tropopause. We therefore suppose a similar sort of relation to hold between the depth and the lapse rate in the case of Jupiter's atmosphere, and that the temperature, even at great depths, can never rise above 300° , to which value we take it to be asymptotic.

The Asymptotic Lapse-rate Model

The simplest relation of this kind is of the form

$$z - z_0 = \frac{\kappa}{300 - T}, \quad (7)$$

where z_0 and κ are constants, which we choose so that, when T is 150° , z is zero, and the lapse rate $\frac{dT}{dz}$ is equal at this level to the adiabatic rate.

This formula is, of course, entirely empirical, and will not, for instance, extrapolate backwards, that is upwards in the atmosphere, where at comparatively small heights we should become involved in negative temperatures; but it will serve our purpose, as the values derived from it cannot differ greatly from those given by any other relation designed to represent a declining lapse rate and a maximum temperature that still admits of the planet being regarded as "cold."

Differentiating (7), we have

$$\frac{dT}{dz} = \frac{(300 - T)^2}{\kappa}, \quad (8)$$

while (4) gives for the adiabatic rate

$$\frac{dT}{dz} = 3.12 \times 10^{-5} \mu \left(1 - \frac{1}{\gamma} \right). \quad (9)$$

Equating the right-hand sides of (8) and (9), and putting T equal to 150, we obtain

$$\kappa = \frac{7.2 \times 10^8}{\mu \left(1 - \frac{1}{\gamma} \right)}, \quad (10)$$

whence

$$z_0 = \frac{-4.8 \times 10^6}{\mu \left(1 - \frac{1}{\gamma}\right)}, \quad (11)$$

so that

$$\begin{aligned} \frac{dP}{dz} &= g_J \rho \\ &= \frac{g_J \mu P}{RT} \\ &= \frac{3.15 \times 10^{-5} \mu P (z - z_0)}{300(z - z_0) - \kappa} \end{aligned} \quad (12)$$

on substitution from (7).

Giving z_0 and κ their values (11) and (10) and simplifying, we have

$$\frac{dP}{P} = 1.05 \times 10^{-7} \mu \left(1 + \frac{2.4 \times 10^6 \mu^{-1} \left(1 - \frac{1}{\gamma}\right)^{-1}}{z + 2.4 \times 10^6 \mu^{-1} \left(1 - \frac{1}{\gamma}\right)^{-1}} \right) dz; \quad (13)$$

whence

$$\log_e P = 1.05 \times 10^{-7} \mu \left\{ z + \frac{2.4 \times 10^6}{\mu \left(1 - \frac{1}{\gamma}\right)} \log_e \left(z + \frac{2.4 \times 10^6}{\mu \left(1 - \frac{1}{\gamma}\right)} \right) \right\} + \text{a constant}, \quad (14)$$

giving

$$\log_e \frac{P}{P_0} = 1.05 \times 10^{-7} \mu z + \frac{2.5 \times 10^{-1}}{1 - \frac{1}{\gamma}} \log_e \left\{ 1 + 4.2 \times 10^{-7} \mu \left(1 - \frac{1}{\gamma}\right) z \right\}. \quad (15)$$

In fig. 3 are shown the pressure-depth curves deduced from the relation (15): *

- (a) For an atmosphere of methane;
- (b) For an atmosphere composed of three parts by weight of hydrogen to four of methane, of which the mean molecular weight is therefore equal to that of helium;
- (c) For an atmosphere of six parts by weight of hydrogen to one of methane.

The figures in brackets at the side of the curves are again the densities.

It will be seen that in (a) the density of the solid state is reached at less than 35 km., in (b) at less than 145 km., and in (c) at 200 km. For an atmo-

* The specific heat of hydrogen is so much larger than that of methane that, for the mixtures, γ comes out at almost exactly 1.4, which is the value that has been used in computing the curves.

sphere of pure hydrogen this would occur at about 270 km., while, if extrapolation beyond this point has any meaning, the hydrogen atmosphere would attain the mean density of the planet (1.34) at a depth of less than 350 km.

The third model thus allows a greater depth than either the adiabatic or the isothermal, but still limits the extent to a fraction of 1 per cent. of the

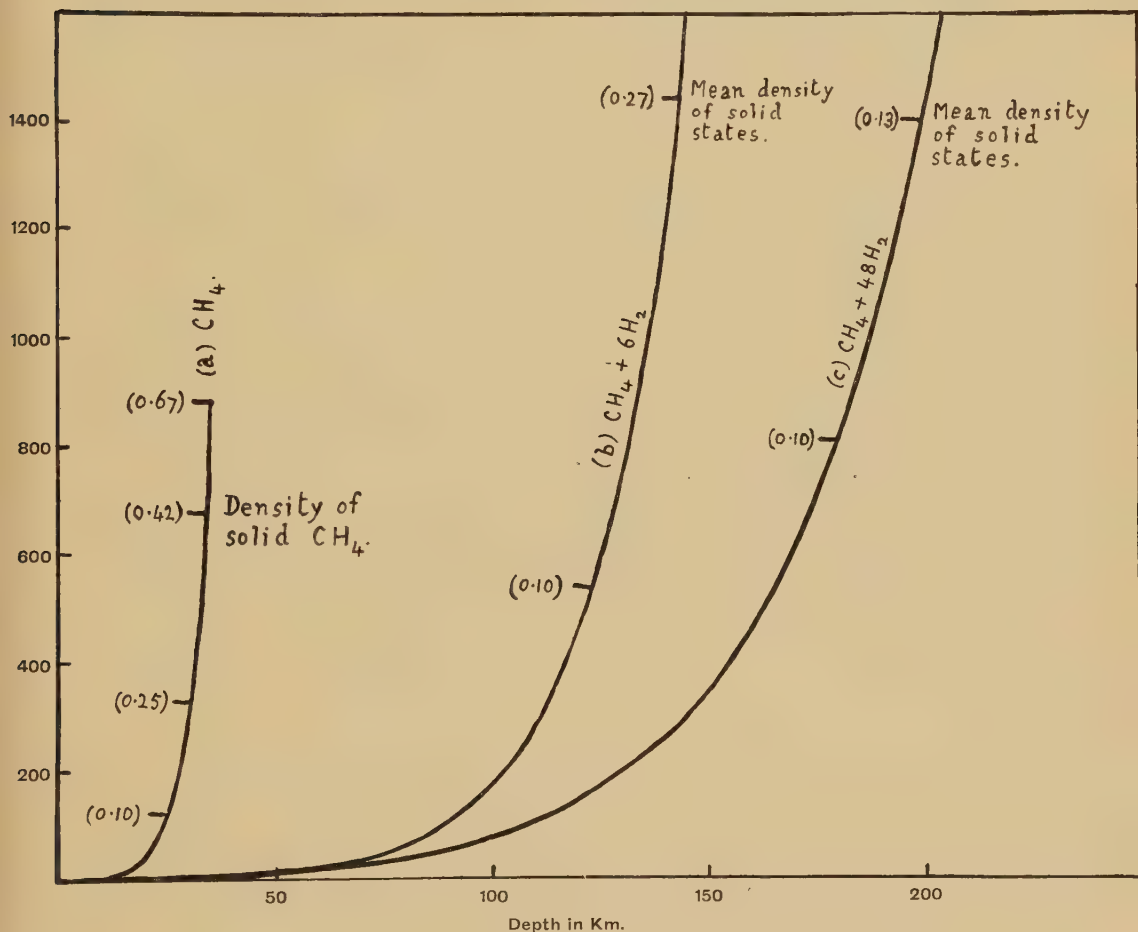


FIG. 3.—The Asymptotic Lapse Rate Model.

planet's radius. Moreover, if either curves (b) or (c) are to be considered relevant, some explanation must be found for the observed abundance of methane in the upper strata of an atmosphere, of which hydrogen is such an important constituent.

It is beyond the scope of this paper to consider the physical properties of atmospheres of great density. Some of these have already been dealt with by Wildt.* Its main purpose is to draw the attention of those who

* *Loc. cit.*

may be desirous of investigating the meteorology of Jupiter to the fact that it is very unlikely that conditions, properly to be described as meteorological, are to be found at greater depths than the first few kilometres below the visible surface. The atmosphere, if it extends below, say, 25 km., would probably become unrecognisable as such by the meteorologist.

It may be noted that the idea of irregular opacity at high altitudes in the atmosphere, put forward by Professor R. A. Sampson to explain departures from the computed times in the observations of satellite phenomena, must be abandoned in the light of the results that have now been obtained.

Summary

The relations between depth, pressure, temperature and density in Jupiter's atmosphere are investigated. Three models are considered :—

- (1) The adiabatic.
- (2) The isothermal.
- (3) A compromise, in which an empirical relation is assumed between the depth and the lapse rate.

The results, for an atmosphere of methane, are plotted in the case of the first two models ; for the last, three curves are given, showing the relations for methane in the pure state, and with the addition of two different proportions of hydrogen.

It is found that the third model admits of greater depths than either of the first two, but that even in this case the great densities that are rapidly attained suggest that the atmosphere must lose its familiar characteristics below a depth of about 25 km.

MODES OF RADIAL OSCILLATION.

T. E. Sterne, Ph.D.

(Communicated by Dr. Harlow Shapley)

1. Introductory

We consider oscillations of a stellar mass in which the motion is purely radial, and such that elements of matter which are at any time equally distant from the centre remain equally distant from the centre. The most general type * of motion, in which the displacement of an element of matter initially at (θ_0, ϕ_0, r_0) is not a function merely of r_0 and the time, but depends besides upon θ_0 or ϕ_0 or both, must in all probability be associated with a considerably greater degree of damping than that which is associated with the spherically symmetrical type of oscillation. The former involves ordinary fluid, as well as radiative, viscosity, while the latter involves only radiative

* See Rosseland, *Publications of the University Observatory, Oslo*, No. 1, 1931 ; and No. 2, 1932.

viscosity; and therefore in all probability the former is more difficult to maintain.

The most general motion of the type which we consider is not periodic in the time. For any particular density distribution there is, however, an infinite set of possible modes of motion, differing from one another in their disposition of nodal surfaces and in their generally incommensurable periods, but each a periodic radial oscillation. When a single mode is excited, the displacement of any particular element of matter is a periodic function of the time; in general it is not a simple harmonic displacement unless the amplitude vanishes. Being periodic, the displacement can be represented by a Fourier series in the time; the various terms in the series may be called "harmonics." The harmonics, unlike the modes, have no independent physical existence. Their origin is purely mathematical; they arise merely because of an attempt to represent a periodic function (which is not a sine or a cosine function) of the time by a Fourier series.* Eddington has shown† that there should be asymmetry when the amplitude does not vanish; asymmetry is an observed property of Cepheid radial-velocity curves. It is in the nature of a mode that it should not be simply harmonic unless it has a vanishing amplitude; and when harmonic analysis reveals in light‡ or radial-velocity curves periods that are submultiples of the longest period, their existence is more of mathematical than of physical interest, for these commensurable periods are in all probability merely those of higher harmonics. The distinction between "mode" and "harmonic" is clear, although the word "harmonic" has often been made to do double duty in the past. We shall be concerned with modes of spherically symmetrical and purely radial oscillation.

The differential equation for small radial oscillations has been given by Eddington;§ we here derive it briefly. We start with the equation of motion, which is in general

$$\ddot{\xi} = -\frac{g_0 \xi_0^2}{\xi^2} - \frac{1}{\rho} \frac{dP}{d\xi}, \quad (1.0)$$

where ξ , g , ρ and P denote distance from centre, gravity, density and pressure respectively. Let the undisturbed values be ξ_0 , g_0 , ρ_0 and P_0 . Define quantities ξ_1 , ρ_1 , P_1 by the equations

$$\xi - \xi_0 = \xi_0 \xi_1; \quad \rho - \rho_0 = \rho_0 \rho_1; \quad P - P_0 = P_0 P_1. \quad (1.1)$$

* The asymmetry of a pure mode may be said to arise from the presence in it of higher harmonics than the lowest, but the statement is a truism; the cause of the asymmetry lies in the equations governing the motion.

† Eddington, *M.N.*, 79, 183, 1919. See also in this connection H. A. Kluyver, *B.A.N.*, No. 268, p. 265, 1935, and *B.A.N.*, No. 276, p. 313, 1936. Woltjer, in *M.N.*, 95, 260, 1935, has discussed the effects upon the higher harmonics of an approximate commensurability between two of the modes, and his methods have been used by Miss Kluyver in her second paper here referred to.

‡ If there were no higher harmonics in a true light-curve, the magnitude curve would of course contain them. They can be created by mere mathematical transformations.

§ Eddington, *Internal Constitution of the Stars*, pp. 186 *et seq.*, Cambridge University Press, 1926.

If the period of the pulsation is $2\pi/n$, than ξ_1 , ρ_1 and P_1 will contain the factor $\cos nt$. During the motion it must be true that

$$\rho_0 \xi_0^2 d\xi_0 = \rho \xi^2 d\xi, \quad (1.2)$$

which, with (1.1), allows us to write (1.0) in the form

$$\ddot{\xi} = -g_0(1 - 2\xi_1) - \frac{1 + 2\xi_1}{\rho_0} \frac{d(P_0 + P_0 P_1)}{d\xi_0} \quad (1.3)$$

correctly to terms of the first order in small quantities. From (1.1) and (1.2)

$$\rho_1 = -(3\xi_1 + \xi_0 \xi_1') \quad (1.4)$$

correctly to such terms; the dash denotes differentiation with respect to ξ_0 . It is assumed that the changes are adiabatic; then $P \propto \rho^\gamma$; hence $P_1 = \gamma \rho_1$; and so from (1.4)

$$P_1 = -\gamma(3\xi_1 + \xi_0 \xi_1'). \quad (1.5)$$

Equation (1.3) breaks up into

$$g_0 + \frac{1}{\rho_0} \frac{dP_0}{d\xi_0} = 0, \quad (1.6)$$

which is the equation of hydrostatic equilibrium satisfied by the undisturbed values, and

$$n^2 \xi_0 \xi_1 + 4g_0 \xi_1 - \frac{1}{\rho_0} \frac{dP_0 P_1}{d\xi_0} = 0; \quad (1.7)$$

which is correct to terms of the first order in small quantities. If we now make use of equation (1.5), we find that after some reduction (1.7) becomes

$$\xi_1'' + (4 - \mu) \frac{\xi_1'}{\xi_0} + \frac{\rho_0}{P_0} \left(\frac{n^2 \xi_0}{\gamma} - \alpha g_0 \right) \frac{\xi_1}{\xi_0} = 0, \quad (1.8)$$

where $\mu = g_0 \rho_0 \xi_0 / P_0$ and where $\alpha = 3 - (4/\gamma)$. This is Eddington's result, valid for small oscillations. The quantity γ must for stability exceed $4/3$; it does not exceed $5/3$ for any materials now known. Hence α must probably lie between 0 and $3/5$. It is possible that γ may vary during a large adiabatic pulsation; but for such a pulsation (1.8) does not hold, and we shall consider small pulsations. It is likely that γ varies somewhat from centre to surface, and certain that the adiabatic assumption itself fails sufficiently near to the surface; but the general character of our results to follow can hardly be seriously invalidated if, for the sake of mathematical simplicity, we treat γ , and hence α , as a constant.

As has already been mentioned, (1.8) does not hold for large amplitudes of oscillation; in its derivation terms of the second and higher orders in small quantities have been ignored. Equation (1.8) is linear in ξ_1 , and different solutions must be additive. But with large amplitudes, the terms of second and higher orders cause the appropriate equation to be non-linear; two of the consequences are that higher harmonics are required in the solutions, and that different solutions are no longer additive. From the

latter it follows that two modes will "interact" if they exist simultaneously (unless the amplitudes are small), and that neither will be strictly periodic.

At the surface the pressure must vanish; P_0 must vanish and P_1 must be finite. It follows from (1.6) and (1.7) that

$$\frac{d(P_0 P_1)}{dP_0} = - \left(4 + \frac{n^2 \xi_0}{g_0} \right) \xi_1,$$

and thence, because of the vanishing of P_0 and $P_0 P_1$ at the surface, that just inside the surface

$$P_1 = - \left(4 + \frac{n^2 R}{g_0} \right) \xi_1, \quad (1.9)$$

where R is the radius. Equation (1.9) is the boundary condition of the pulsation.

Eddington * has obtained the period of the zero'th, and Edgar † the period of the first mode corresponding to the density distribution of an Emden polytrope of index 3; they had to resort to numerical integrations to reach their results. We believe it to be of importance to investigate in detail the nature of the radial modes of as many different stellar models as possible. If ρ_0 is a known function of ξ_0 , then P_0 and g_0 are likewise known functions of ξ_0 since they may be computed from gravitational theory. Accordingly, the nature of the small radial modes of any stellar model depends upon only two things: (1) the relation between ρ_0 and ξ_0 , and (2) the value of γ . We shall here study the small radial oscillations of three different density distributions, and for each one we shall obtain an algebraic expression for the periods of all its modes. We number the modes 0, 1, 2, 3, . . . etc.

Before proceeding further we write equations (1.8) and (1.9) in more convenient forms. Let $\xi_0 = Rx$. Then (1.8) becomes

$$\xi_1'' + \frac{4 - \mu}{x} \xi_1' + \frac{R \rho_0}{P_0} \left(\frac{n^2 R}{\gamma} - \frac{\alpha g_0}{x} \right) \xi_1 = 0; \quad (1.91)$$

and the boundary condition (1.9) becomes, from (1.5),

$$\left(\frac{n^2 R}{\gamma g_0} - \alpha \right) \xi_1 = \xi_1' \quad (1.92)$$

at $x = 1$. Here dashes denote differentiation with respect to x , a convention henceforth to be followed.

2. Model 1: The Modes of a Sphere of Uniform Density

Consider a sphere in which the undisturbed density is constant and equal to the mean density, $\bar{\rho}$. The mass within any radius is $\frac{4}{3}\pi\bar{\rho}\xi_0^3$; the undisturbed values of gravity and the pressure are $g_0 = (4\pi/3)G\xi_0\bar{\rho}$ and

* Eddington, *loc. cit.*

† Edgar, *M.N.*, **93**, 422, 1933.

$P_0 = (2\pi/3)G\bar{\rho}^2(R^2 - \xi_0^2)$ respectively. G is the gravitational constant. The quantity $\mu = 2x^2/(1 - x^2)$, therefore, and substituting these values in equations (1.91) and (1.92) we find for the differential equation satisfied by ξ_1 ,

$$(1 - x^2)\xi_1'' + \frac{4 - 6x^2}{x}\xi_1' + \mathcal{J}\xi_1 = 0, \quad (2.0)$$

and for the boundary condition at $x = 1$,

$$\mathcal{J}\xi_1 = 2\xi_1', \quad (2.05)$$

where the number

$$\mathcal{J} = \frac{3n^2}{2\pi G\bar{\rho}\gamma} - 2a. \quad (2.06)$$

Equation (2.0) has regular singularities at the origin and at $x = 1$. Since the equation is linear, we may arbitrarily set $\xi_1(0) = 1$. We must find regular integrals which are finite in the range $0 \leq x \leq 1$, and which also satisfy the boundary condition (2.05). These solutions will in general exist only for certain values of $\mathcal{J}$, which we must find. The roots of the indicial equation are -3 and 0 ; we must choose the latter and expand the solution

about the origin in a power series of the form $\xi_1 = \sum_0^\infty a_k x^{k/5}$ where $a_0 = 1$.

It is found by substitution that the terms in odd powers of x vanish, and that the coefficients of the even terms satisfy the recurrence formula

$$a_{k+2} = a_k \frac{k^2 + 5k - \mathcal{J}}{(k+2)(k+5)}. \quad (2.07)$$

The boundary condition (2.05) is obviously satisfied if ξ_1'' remains finite at $x = 1$, because of the form of (2.0). Hence all the conditions will be satisfied if we choose $\mathcal{J}$ so as to make the series solution terminate with some term, say the $2j$ 'th where j is zero or any positive integer. This it will do, by (2.07), if

$$\mathcal{J} = 2j(2j+5). \quad (2.08)$$

This is a sufficient condition; it is also necessary. For let $\mathcal{J}$ have any value not given by (2.08). Then the series expansion of $\xi_1' = \sum_0^\infty k a_k x^{k/5}$, by (2.07), never terminates; the coefficients are ultimately all of the same sign and the ratio $ka_k/a_{k+2}(k+2)$ approaches unity as $k \rightarrow \infty$. The series therefore converges when $x < 1$. When $x = 1$, however,

$$\lim_{k \rightarrow \infty} \frac{k}{2} \left\{ \left| \frac{a_{k+2}(k+2)}{a_k k} \right| - 1 \right\} = 0$$

by (2.07); and hence the series can be shown to diverge.* Hence it is impossible for the boundary condition (2.05) to be satisfied, and at the

* By an obvious extension of the argument given by Whittaker and Watson, *Modern Analysis*, chapter ii, section 2.37, Cambridge University Press, 1927.

same time for ξ_1 to be finite, at $x = 1$. The values of $\mathcal{J}$ given by (2.08) are therefore the only values that lead to permissible solutions.

We have therefore found all the radial modes of this model. The first few values of $\mathcal{J}$ are 0, 14, 36, 66, 104, 150, 204. The first few solutions are

$$\begin{array}{lll} j=0; & \mathcal{J}=0; & \xi_1=1 \\ j=1; & \mathcal{J}=14; & \xi_1=1-(7/5)x^2 \\ j=2; & \mathcal{J}=36; & \xi_1=1-(18/5)x^2+(99/35)x^4 \\ j=3; & \mathcal{J}=66; & \xi_1=1-(33/5)x^2+(429/35)x^4-(143/21)x^6, \end{array}$$

etc., easily obtained from (2.07). An examination reveals that ξ_1 vanishes between $x=0$ and $x=1$ at j different values of x ; these points and the origin are nodes of the displacement. The period of the j 'th mode is $2\pi/n$, where

$$n^2 = \frac{2\pi G \bar{\rho} \gamma}{3} (\mathcal{J} + 2\alpha), \quad (2.09)$$

in which $\mathcal{J}$ is given by (2.08).

There are a number of observations to make at this point. Firstly, if the matter within a homogeneous sphere in hydrostatic equilibrium is given, adiabatically, any known spherically symmetrical system of small and purely radial displacements, and is then at once released, it is possible to represent this initial displacement by a linear combination of the ξ_1 polynomials which we have found, and then to predict the course of the subsequent motion. Secondly, when j and therefore $\mathcal{J}$ are zero, and α is zero, the period by (2.09) is infinite, showing that the model is in neutral equilibrium with respect to the zero'th mode; while if α is negative, which happens when γ is less than $4/3$, the period is imaginary, showing that the model is then unstable with respect to the zero'th mode. On the other hand, the j 'th mode is stable when

$$\mathcal{J} + 2\alpha > 0,$$

which is equivalent to

$$\gamma > \frac{4}{j(2j+5)+3}.$$

Instability cannot arise from the higher modes, therefore, even when γ is considerably smaller than $4/3$; the first mode is stable when γ exceeds $2/5$, and the second when γ exceeds $4/21$. Instability must arise from the zero'th mode first, as γ is decreased.

Finally, it is seen from (2.09) that the periods of successively higher modes are altered by diminishing (as well as by relatively diminishing) amounts through changes in α , but that the period of the zero'th mode is fairly sensitive to α . The ratio of the periods of the zero'th and first modes is $(14+2\alpha)^{1/2}/(2\alpha)^{1/2}$ and can be varied all the way from ∞ to 3.559 by varying α from 0 to 0.6. The ratio of the periods of the first and second modes, on the other hand, varies only from 1.604 to 1.564 as α varies from 0 to 0.6, and of the second and third only from 1.354 to 1.344. The effect of an increase in α is always to diminish the period.

3. *Model 2: The Modes of a Sphere in which the Density varies Inversely as the Square of the Distance to the Centre*

We next consider a sphere more highly concentrated towards the centre, in which the undisturbed density varies as ξ_0^{-2} . We have $\rho_0 = R^2 \bar{\rho} / 3 \xi_0^2$; the mass within any distance is $(4\pi/3)R^2 \bar{\rho} \xi_0$; the undisturbed values of gravity and pressure are $g_0 = 4\pi G R^2 \bar{\rho} / 3 \xi_0$ and $P_0 = (2/9)\pi G R^4 \bar{\rho}^2 (\xi_0^{-2} - R^{-2})$, respectively. The law of density in the form in which we take it leads to an infinite density at the centre; we may imagine, however, that the law holds only to a very small distance from the centre, within which distance the density is constant; this avoids the physical difficulty without appreciably altering either the mass within any significant radius, or the values of gravity and pressure at significant radii (since the mass within any radius varies as the radius). We have $\mu = 2/(1 - x^2)$, and for this model equations (1.91) and (1.92) become

$$(1 - x^2)\xi_1'' + \frac{2 - 4x^2}{x}\xi_1' + (\mathfrak{J}x^2 - 2\alpha)\frac{\xi_1}{x^2} = 0, \quad (3.0)$$

and, at $x = 1$,

$$(\mathfrak{J} - 2\alpha)\xi_1 = 2\xi_1' \quad (3.05)$$

where the number

$$\mathfrak{J} = \frac{3n^2}{2\pi G \gamma \bar{\rho}}. \quad (3.06)$$

Equation (3.0) has regular singularities at the origin and at $x = 1$. We wish to find those values of $\mathfrak{J}$, if there are any, for which (3.0) has regular integrals finite in the range $0 \leq x \leq 1$, and which also satisfy (3.05). The roots of the indicial equation are $\frac{1}{2}[-1 \pm (1 + 8\alpha)^{\frac{1}{2}}]$; we must choose the larger one and expand the solution about the origin in a power series of the form $\xi_1 = x^q \sum_{k=0}^{\infty} a_k x^k$ where $a_0 = 1$, and where

$$q = \frac{1}{2}[-1 + (1 + 8\alpha)^{\frac{1}{2}}]. \quad (3.065)$$

It is found by substitution that the terms corresponding to odd values of k vanish; and that the coefficients of the even terms satisfy the recurrence formula

$$a_{k+2} = a_k \frac{(q+k)(q+k+3) - \mathfrak{J}}{(q+k+2)(q+k+3) - 2\alpha}. \quad (3.07)$$

The boundary condition (3.05) is obviously satisfied if ξ_1'' remains finite at $x = 1$, because of the form of (3.0). All the conditions are satisfied if $\mathfrak{J}$ has any of the values

$$\mathfrak{J} = (2j+q)(2j+q+3), \quad (3.08)$$

where j is zero or any positive integer, for the series then terminates with the j 'th even term. It is easy to show, by a repetition of the argument given in the last section, that the condition (3.08) is necessary as well as sufficient; if $\mathfrak{J}$ has any value not given by (3.06) the series for ξ_1 never

terminates, and leads to an infinite value for ξ_1' at $x=1$, making it impossible for (3.05) to be satisfied and at the same time for ξ_1 to be finite at $x=1$.

We have therefore found all the radial modes of this model. The period of the j 'th mode is $2\pi/n$, where

$$n^2 = \frac{2\pi G\gamma\bar{\rho}}{3} \mathcal{J} \quad (3.09)$$

in which $\mathcal{J}$ is given by (3.08) and q by (3.065). Unlike the polynomials which were found for the uniform model, the ξ_1 polynomials here depend upon the value of α ; we do not list them. They are easily written down from the recurrence formula. It is found that for the j 'th mode there are j nodes in the displacement between the origin and the boundary, with an additional node at the origin.

As for the uniform model, any small, arbitrarily started, radial, and spherically symmetrical pulsation can be represented by a linear combination of the ξ_1 polynomials given by (3.07). When α is zero then the zero'th mode has an infinite period and the model is then in neutral equilibrium with respect to the zero'th mode; for negative values of α the model is unstable with respect to the zero'th mode. When α is zero, the higher modes are stable, since $\mathcal{J}=2j(2j+3)$ is then positive. When $\alpha < 0$, q becomes negative; and ξ_1 is infinite at $x=0$. It is difficult to say what meaning is to be attached to this in connection with stability; the higher modes are still stable in the sense that the period is real and finite as long as α remains larger than $-1/8$, and thus as long as $\gamma > 32/25$. For smaller values of α and γ the motion is unstable for all the modes.

The ratio of the periods of the zero'th and first modes varies all the way from ∞ to 2.432 as α varies from 0 to 0.6; and that of the first and second modes from 1.673 to 1.533. The periods of successively higher modes are altered by relatively diminishing amounts through changes in α , while the period of the zero'th mode is fairly sensitive to α . The period is a diminishing function of α .

4. Model 3: *The Modes of a Sphere in which nearly all the Mass is contained in a Particle at the Centre*

In Model 1 the undisturbed central density equals the mean density; in Model 2 the ratio of the undisturbed central density to the mean density may approach ∞ . In the Emden polytrope of index 3 the central density is 54.36 times the mean density. Nevertheless, in a certain sense the Emden polytrope is more highly concentrated towards the centre than either of the preceding models; for in Model 1, $1/8$ of the total mass is contained in a sphere of half the total radius; in Model 2, $1/2$ of the mass is contained in such a sphere; while in the polytrope of index 3 nearly $9/10$ of the mass is contained in such a sphere. We therefore consider Model 3 as a limiting case (one of many that could be considered) in which nearly all the mass is contained inside of any sphere of non-vanishing radius. This model is chosen to consist of a discrete particle of mass $(4\pi/3)R^3\bar{\rho}[1-(3a/R^2\bar{\rho})]$ at

the centre, plus a continuous density distribution $a\xi_0^{-2}$ at all points inside the boundary, where a is a constant which we shall presently allow to approach zero. The mass within ξ_0 is

$$(4\pi/3)R^3\bar{\rho}[1 - (3a/R^2\bar{\rho})] + 4\pi a\xi_0,$$

and thus

$$g_0 = (4\pi/3)R^3G\bar{\rho}[1 - (3a/R^2\bar{\rho})]\xi_0^{-2} + 4\pi aG\xi_0^{-1}$$

and

$$P_0 = (4\pi/9)R^3G\bar{\rho}[1 - (3a/R^2\bar{\rho})]a(\xi_0^{-3} - R^{-3}) + 2\pi a^2G(\xi_0^{-2} - R^{-2}).$$

Now as $a \rightarrow 0$, ρ_0 and P_0 separately approach 0 except at the centre, while

$$g_0 \rightarrow (4\pi/3)R^3G\bar{\rho}\xi_0^{-2};$$

the ratio

$$(\rho_0/P_0) \rightarrow 9x/4\pi R^2G\bar{\rho}(1 - x^3),$$

however, and thus

$$\mu \rightarrow 3/(1 - x^3),$$

while

$$(R^2\rho_0n^2/P_0\gamma) \rightarrow 9n^2x/4\pi\gamma G\bar{\rho}(1 - x^3)$$

and

$$(aR\rho_0g_0/P_0x) \rightarrow 3a/x^2(1 - x^3).$$

Accordingly, in the limit the differential equation (1.91) becomes

$$(1 - x^3)\xi_1'' + \frac{1 - 4x^3}{x}\xi_1' + \left(\mathcal{J}x - \frac{3a}{x^2}\right)\xi_1 = 0, \quad (4.0)$$

and the boundary condition (1.92) becomes, at $x = 1$,

$$(\mathcal{J} - 3a)\xi_1 = 3\xi_1', \quad (4.05)$$

where the number

$$\mathcal{J} = \frac{9n^2}{4\pi G\gamma\bar{\rho}}. \quad (4.06)$$

Equation (4.0) has regular singularities at $x = 0$ and at $x = 1$; the roots of the indicial equation are $\pm(3a)^{\frac{1}{2}}$; we expand the solution about the origin in a series of the form $\xi_1 = x^a \sum_0^\infty a_k x^{k/2}$, where $a_0 = 1$ and where

$$q = +\sqrt{3a}. \quad (4.065)$$

We find by substitution that the only non-vanishing coefficients are those for which $k = 3j$, where j is zero or a positive integer, and that the coefficients obey the recurrence formula

$$a_{k+3} = a_k \frac{(k+q)(k+q+3) - \mathcal{J}}{(k+3+q)^2 - 3a}. \quad (4.07)$$

Hence the solution is finite in the range $0 \leq x \leq 1$, and the boundary condition is satisfied, if

$$\mathcal{J} = (3j+q)(3j+q+3), \quad (4.08)$$

in which case the solution is a polynomial; and it can be shown as usual

that permissible solutions can be obtained for no other values of j than those given by (4.08), which therefore furnishes all the radial modes. The period of the j 'th mode is $2\pi/n$, where

$$n^2 = \frac{2\pi G\gamma\bar{\rho}}{3} \left(\frac{2}{3}j\right), \quad (4.09)$$

in which j is given by (4.08) and q by (4.065).

For Model 3, a must not be smaller than 0 or all the modes are unstable ; when $a=0$ all modes are stable except the zero'th. The ratio of the periods of the zero'th and first modes varies from ∞ to 2.339 as a varies from 0 to 0.6, the ratios between the periods of consecutive and successively higher modes are diminishingly sensitive to the value of a , and the periods of successively higher modes are diminishingly sensitive to a . As usual, the period of any particular mode is a diminishing function of a .

5. The Polytrope of Index 3. Numerical Comparisons

As has been mentioned previously, Eddington and Edgar have investigated by numerical integration some of the radial modes of a polytrope of index 3. Eddington * lists the values of ω^2 for the zero'th mode corresponding to values of a equal to 0.1, 0.2 and 0.6 ; Edgar † the values of ω^2 for the zero'th mode when a is 0.4, and for the first mode when a has the values 0.1 and 0.4. From Eddington's equation immediately following his equation (130.2) on page 192, *loc. cit.*, it follows that

$$n^2 = \frac{2\pi G\gamma\bar{\rho}}{3} (54.36) \left(\frac{3}{2}\right) \omega^2.$$

The values of ω^2 found by Eddington and Edgar ‡ were :—

$\begin{array}{c} j \\ a \end{array}$	0	1
0.1	0.0315	0.30
0.2	0.060	...
0.4	0.110	0.466
0.6	0.156	...

* Eddington, second reference.

† Edgar, *loc. cit.*

‡ It is perhaps worth noting that these figures are not strictly comparable to ours, owing to the circumstance that Eddington and Edgar chose as the boundary condition in their numerical integrations not the formally rigorous one that P_0P_1 must vanish at the boundary, but the approximate one that P_1 vanishes somewhere inside the boundary. The former condition, which we have used, is invalidated in an actual star by the failure of the adiabatic assumption at the surface, although the results are probably not seriously in error on that account ; the latter condition is completely false for the uniform model (which in the zero'th mode has no node at all in P_1) and involves an element of arbitrariness in all cases. Edgar's value $\omega^2=0.466$ for $a=0.4$ and $j=1$ has in particular been challenged by Miss Kluyver (second reference). We merely quote here, with the above comments, the results obtained by Eddington and Edgar.

We may summarize our own results, along with those of Eddington and Edgar, as follows. The period of the j 'th mode is $2\pi/n$, where

$$n^2 = \frac{2\pi G\gamma\bar{\rho}}{3} A \quad (5.0)$$

in which A has the following values :—

For Model 1 : $A = 2j(2j+5) + 2a$.

For Model 2 : $A = (2j+q)(2j+q+3)$; $q = \frac{1}{2}(-1 + \sqrt{1+8a})$.

For Model 3 : $A = (2/3)(3j+q)(3j+q+3)$; $q = +\sqrt{3a}$.

Polytrope 3 : A has the following known values :—

$j \backslash a$	0	1
0.1	2.57	24.5
0.2	4.89	...
0.4	8.97	38.0
0.6	12.7	...

From (5.0) it follows that

$$T \sqrt{\frac{G\bar{\rho}}{6\pi}} = \frac{1}{\sqrt{\gamma A}} \quad (5.1)$$

where T is the period. We list values of $T(G\bar{\rho}/6\pi)^{\frac{1}{2}}$ for the four models, for each of four values of a , and for the first few modes :

$$a = 0.1$$

$j \backslash$ Model	0	1	2	3
1	1.904	0.2260	0.1415	0.1046
2	1.157	0.2541	0.1557	0.1132
3	0.748	0.2164	0.1319	0.0953
Polytrope	0.531	0.1720	...	...

$$a = 0.2$$

$j \backslash$ Model	0	1	2	3
1	1.323	0.2205	0.1387	0.1027
2	0.831	0.2392	0.1492	0.1092
3	0.599	0.2026	0.1259	0.0917
Polytrope	0.378	...	...	...

$\alpha = 0.4$

j Model	0	1	2	3
1	0.901	0.2096	0.1329	0.0986
2	0.593	0.2159	0.1382	0.1023
3	0.466	0.1832	0.1167	0.0859
Polytrope	0.269	0.1308	...	...

$\alpha = 0.6$

j Model	0	1	2	3
1	0.707	0.1987	0.1270	0.0945
2	0.480	0.1972	0.1287	0.0960
3	0.393	0.1680	0.1089	0.0808
Polytrope	0.217	...	...	...

The ratio of the period of the first mode to that of the zero'th may also be of interest. It is as follows :—

a Model	0.1	0.2	0.4	0.6
1	0.119	0.167	0.233	0.281
2	0.220	0.288	0.364	0.411
3	0.289	0.338	0.393	0.427
Polytrope	0.324	...	0.486	...

One notices that although for the zero'th mode Models 1, 2, 3 and the polytrope give successively smaller periods for the same mean density and value of γ , the order is different for the higher modes. The polytrope, however, always yields the smallest periods, and yields the largest ratio between the period of the first mode and that of the zero'th mode. This ratio appears to increase regularly for Models 1, 2, 3 and the polytrope.

Although the first mode of the polytrope was found by Edgar to suffer more from radiative damping than the zero'th mode, the difference was not very great, and the resulting times of decay of the free vibrations were of the same order of magnitude. One might perhaps expect that the zero'th mode would be the easiest to maintain, and that in most pulsating stars the zero'th mode would be the most prominent. It might also be expected, however, that a few pulsating stars would show evidence of higher modes superimposed on the zero'th. One's belief in the latter possibility is strengthened by the consideration that the dependence of ρ_1 , and hence of the range of temperature, upon ξ_0 varies from mode to mode; so that special distributions of the sources might tend to strengthen, in some stars, the excitation of one or more of the higher modes.

BRIGHT SOLAR ERUPTIONS AND RADIO FADINGS DURING THE YEARS 1935-36.

H. W. Newton and H. J. Barton.

(Communicated by the Astronomer Royal)

1. During the past eighteen months attention has been drawn by various observers * to occasional coincidences between bright eruptions on the Sun and sudden fadings in short-wave wireless reception experienced over a large part of the Earth. During 1935 such coincidences were not obvious, but in 1936 as the Sun became increasingly active they were numerous enough to suggest a possible relation between the two phenomena. During the last solar minimum (1933-8) both phenomena appear to have been very infrequent. Previous to this minimum epoch, no data exist for the purposes of a statistical comparison. In the present paper comparative data obtained during 1935 and 1936 are discussed, in order to test the suggestion whether a solar-terrestrial relationship does in fact exist between these two phenomena, whose characteristics may first be briefly described.

2. *Bright Solar Eruptions.*—The first bright solar eruption ever recorded was that observed by Carrington and Hodgson on 1859 September 1. This unique eruption was visible in an ordinary telescope, and a repetition by this means has never been observed. The bright eruptions included in this paper are those which have been observed spectroscopically in monochromatic light, corresponding to $H\alpha$ (6563 Å.) and to the lines of ionized calcium, H and K (3969 and 3934 Å.).

In 1870 Young observed the form of a brilliant hydrogen eruption on the Sun's disc, by means of an ordinary solar spectroscope used with a widened slit. Brilliant reversals of $H\alpha$, which denote the presence of bright eruptions, have also been systematically observed by various observers but not published in detail. The spectroheliograph (perfected in 1892), working in H or in K (occasionally in $H\beta$) and later in $H\alpha$, depicted the appearance of bright eruptions when they were caught by chance on the daily spectroheliograms at one observatory or another. In 1926, however, Dr. G. E. Hale introduced his spectrohelioscope, which is admirably adapted for the visual search and observation of bright eruptions in $H\alpha$. The arrangement of an international programme of observations under the auspices of Commission No. 11 of the International Astronomical Union has recently led to a systematic search for their occurrence. Although in 1935 there were gaps in the observing programme, mainly between 21^h-0^h-9^h U.T., these have since been partly filled by new stations at work with spectro-

* Including Richardson, *P.A.S.P.*, 48, 178 and 278, 1936. Newton and Barton, *Journal B.A.A.*, 46, 196, 1936; Newton, *ibid.*, 47, 109, 1937; *Nature*, 138, 1017, 1936. Torreson, Scott and Stanton, *Terrestrial Magnetism*, 41, 199, 1936. McNish, *Nature*, 139, 244, 1937. A recent paper by J. H. Dellinger in *Terr. Mag.*, 42, 49, 1937, will be referred to later; see also *Physical Review*, 50, 1189, 1936.

helioscopes. Cloudy weather is, however, a severe hindrance at several of the participating observatories. Fig. 1 shows the observed monthly frequency of bright eruptions from 1935 to 1936 which may be compared with the sunspot curve given by the Zurich sunspot numbers, and also the frequency of radio fadings given in Tables II and IIA.

Bright solar eruptions have the following general characteristics :—

(1) They appear very suddenly, rise rapidly to a maximum, in 5 to 10 minutes, and die away less quickly. An hour after an eruption has first appeared, all traces of it may have disappeared.

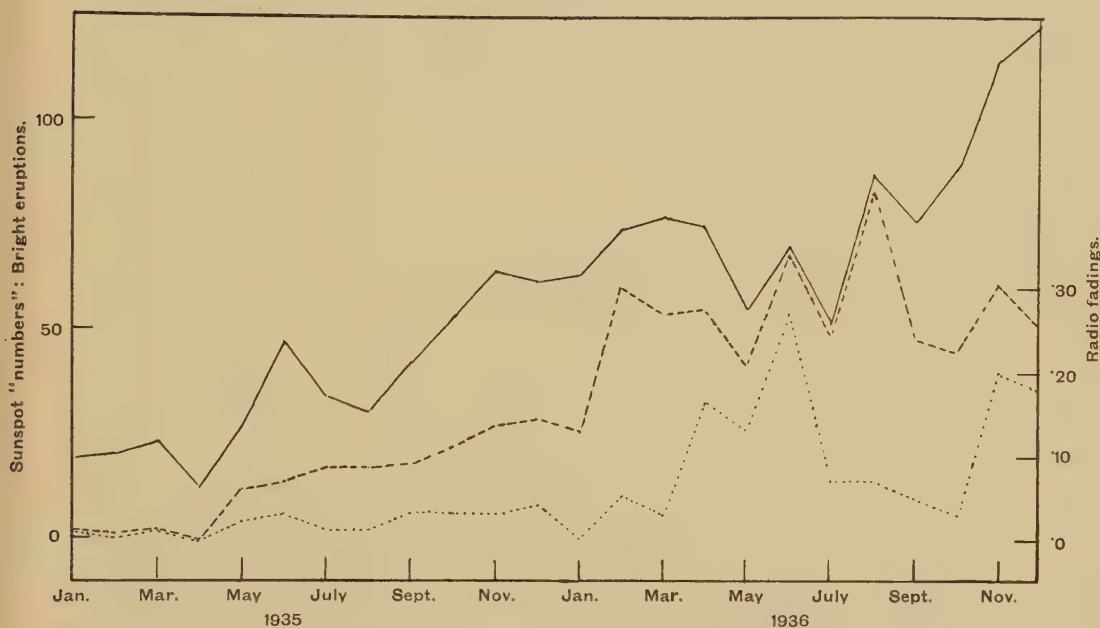


FIG. 1.—Observed monthly frequency, 1935-1936, of
Sunspots ——— Bright H α eruptions - - - - - Radio fadings

(2) Their average duration is from about 20 minutes, for those of lesser intensity, to 40 minutes, for those of greater intensity (1935-36 averages).

(3) In central intensity they sometimes equal or exceed that of the continuous spectrum adjacent to the line in the spectrum in which they are being observed.

(4) They are nearly always associated with sunspots.

(5) The photosphere, including the structure of sunspots, adjacent to the bright eruption appears unaffected.

(6) The eruptions do not appear to be areas of high temperature black-body radiation.*

(7) Occasionally two or more bright eruptions occur nearly simultaneously in widely separated groups of sunspots. Such occurrences are more frequent than can be ascribed to pure chance.†

* *Annual Report of the Mount Wilson Observatory, 1935-36, p. 172.*

† *Loc. cit.*, p. 171.

(8) The areas of the brilliant monochromatic emission range from tiny patches the size of a small sunspot (say 20 millionths of the Sun's hemisphere) to large patches or streaks equal in aggregate area to that of a large sunspot (say 500 millionths).

(9) The brilliant patches, which themselves usually possess no measurable outward velocity, are often closely associated with absorption markings possessing high radial velocity as great as 400–500 km./sec. Outward velocities are often followed by inward velocities indicating that part, at least, of the ejected cloud of gas is falling backwards to the Sun.

Table I gives the number of solar eruptions observed during 1934–36 extracted from the Zurich quarterly *Bulletin for Character Figures of Solar Phenomena* in which all observations made under the I.A.U. Scheme from contributing observatories are tabulated by M. d'Azambuja. The eruptions are arranged according to their observed intensity on an approximate scale of 1 (the smallest and least intense), 2, 3 (the largest and most intense).

TABLE I

Year	Number of Bright Solar Eruptions of Intensity			Total, including Unclassified Cases	Mean Sunspot Number
	1.	2.	3.		
1934	14	1	0	15	8.7
1935	137	18	2	161	36.1
1936	472	134	27	642	80.4

Table IA gives the number of bright eruptions (all classes) observed at a single observatory (Greenwich) with a spectrohelioscope systematically in operation through the last solar minimum. The observations indicate that the rapid rise in the observed frequency of bright eruptions during the last two years is largely real and is not merely due to the increasing number of observatories taking part in recording these eruptions.

TABLE IA

Year	1932	1933	1934	1935	1936
Number of bright eruptions	12	5	5	25	66 *
Mean sunspot number	11	6	9	36	80

* The instrument was being remounted in 1936 January and one month's observations were lost.

3. *Radio Fadings*.—The data of rapid fadings (experienced on short-wave radio transmission) used in this paper are derived from—

(a) A list prepared by the Secretary (Dr. Jouaust) of the French National Committee of the Union Radio Scientifique Internationale, and communicated to the Royal Observatory, Greenwich, by the President (Dr. Rayner) of Commission 1 of the Union. The data give the times of recorded fadings observed in France or in the French Colonies.

(b) A list of fadings, communicated to the Astronomer Royal by the Engineer-in-Chief, Cable and Wireless Ltd., London. These fadings are

those recorded on channels operated by the Company from this country ; the wave-lengths employed usually ranging from 16 to 25 metres according to circumstances.

The combined data (a) and (b) may be taken as being representative of fadings recorded on channels operating from Western Europe.

These radio fadings, which are to be distinguished from those recorded on channels traversing the ionosphere near the auroral zones when magnetic activity is present, have the following characteristics :—*

(1) The drop in signal strength occurs quite suddenly, being sensibly simultaneous on all routes affected.

(2) The duration of the excessive attenuation is approximately 10 to 45 minutes.†

(3) The recovery period is less definite, extending in some instances to one hour or more.

(4) The highest frequencies are in general affected to a lesser degree and for a shorter time.

(5) A sudden increase, on occasions, of noise field at the commencement of, or a minute or two prior to, the fade ; the effect has been described as similar to that observed during charged rain storms, or " Fizzlies".

(6) The phenomenon is observed, in general, on those waves traversing a daylight path.

4. A comparison will now be made between these radio fadings and the data of solar eruptions. Table II contains the following information.

(1) The date and time of the beginning and ending of the radio fading. In those cases in which the fading is reported from both sources the mean times are tabulated. The mean difference of time of commencement in 37 doubly observed cases is about 3 minutes. The ending of the phenomenon is less definite than the commencement, the mean difference in the doubly observed cases being 12 minutes. (2) Particulars of any solar eruption occurring within 30 minutes of the beginning or ending of each recorded fading are next listed as follows. The times during which the eruption was observed, the intensity of the eruption on a scale of 1 to 3 as previously defined, and the angular distance from the Sun's central meridian. Heavy type figures indicate that the actual beginning or end of the eruption was observed. Even in the most favourable circumstances the observed times of commencement are probably systematically late by one or two minutes, since the light intensity of the eruption has to rise above that of the normal bright flocculus to be distinguished by the observer as an eruption. In cases where no eruption appears recorded in the solar *Bulletin* it cannot be inferred from the published data whether the Sun was actually under observation (this information will in the future be available). In six cases, however, some information is available and is given on p. 607 of this paper. Cases marked "No scheduled observations" indicate that gaps occur in the observing programme as given in *Trans. I.A.U.*, 5, 60, 1936.

* From information kindly supplied by the Engineer-in-Chief, Cable and Wireless, Ltd. See also a short article by R. Jouaust in *Onde Électrique*, 16, 211, 1937.

† The mean duration given by those radio fadings in Table II is 23 minutes.

TABLE II

Comparative Data of Radio Fadings and Bright Solar Eruptions, 1935-36
(Radio Data : Union Radio Scientifique Internationale and Cable and Wireless, Ltd.)

Date	Radio Fading		Bright Solar Eruption				Time-interval Fading - Eruption
	From U.T.	To U.T.	From U.T.	To U.T.	Intensity	Distance from C.M.	
1935	h m	h m	h m	h m		o	m
May 12	11 57	12 15	...	...	...	...	...
" 30	12 30	...	...	...	...	...	...
June 14	17 00	...	...	...	...	...	...
" 24	14 00	...	...	...	...	...	...
" 26	18 00	...	...	...	...	...	...
July 6	14 07½	...	...	...	...	...	...
Oct. 15	19 30	...	...	...	...	...	...
" 24	12 00	...	...	...	...	...	...
1936							
Feb. 6	15 20	16 10	...	...	...	...	...
" 8	13 27½	14 30	13 30	14 00	1	72 E.	- 2½
" 14	15 17	15 50	15 30	16 00	2	o	- 13
Apr. 1	09 34	09 55	09 26	10 40	2	30 W.	+ 8
" 1	12 00	12 20	...	...	...	...	...
" 6	13 57	14 25	...	...	...	...	...
" 8	16 49	17 16	{ 16 45 16 47	{ 17 03 18 30	2 3	10 W. 5 W.	+ 4 + 2
" 9	13 40	13 48	13 30	14 30	2	18 W.	+ 10
May 15	06 00	06 30	(07 04	08 30	3	27 E.	- 64) *
" 25	12 34	12 47	...	...	...	...	...
" 26	11 30½	11 43	11 15	12 03	2.5	41 W.	+ 15½
" 28	07 29	07 47	07 15	08 00	2	82 E.	+ 14
" 28	14 05½	14 19	...	...	...	...	...
" 28	18 00	18 20	...	...	...	...	...
" 29	10 24	10 35	Sun under observation : see p. 607				...
June 3	00 45	...	No scheduled observation				...
" 3	16 35	16 55	16 29	17 00	1	o	+ 6
" 4	11 54½	12 06	...	...	...	...	...
" 9	14 25	14 57	14 24	14 32	1	79 W.	+ 1
" 10	14 25	14 40	...	...	...	...	...
" 10	20 55	21 20	...	...	...	...	...
" 11	06 27½	06 42	...	...	...	...	...
" 11	12 29	12 42	...	...	...	...	...
" 16	13 30	13 37	{ 13 25 13 27 06 52	{ 13 48 13 42 07 14	2 ? 1	75 E. 60 E. 58 E.	+ 5 + 3 + 28
" 17	07 20	07 50	{ 07 00 07 27	{ 07 15 08 10	? 1	50 E. 62 E.	+ 20 - 7
" 17	12 48	12 54	...	...	...	...	...
" 19	09 10	09 25	09 00	11 00	2	35 E.	+ 10
" 25	10 47½	11 13	{ 10 51 11 10	{ 11 40 11 30	1 1	62 E. 46 W.	- 3½ - 22½
" 25	11 15	11 25	{ 10 51 11 10	{ 11 40 11 30	1 1	62 E. 46 W.	+ 24 + 5

* See footnote on p. 599.

Date	Radio Fading		Bright Solar Eruption				Time-interval Fading - Eruption
	From U.T.	To U.T.	From U.T.	To U.T.	Intensity	Distance from C.M.	
1936	h m	h m	h m	h m	...	°	m
July 1	13 42	14 15	...	...	...	...	...
" 15	05 05	05 15	No scheduled observation				...
" 15	13 29	13 41	13 30	13 40	2	17 W.	- 1
" 17	12 00	12 30	...	...	...	...	...
" 30	13 40	13 50	...	...	...	...	...
" 31	00 16	00 25	00 11	00 30	3	46 E.	+ 5
" 31	09 30	09 50	...	...	...	...	...
Aug. 23	11 30	12 10	...	...	...	...	...
" 25	18 35	18 58	18 58	19 22	3	66 E.	- 23
" 25	20 10	...	...	...	...	...	...
" 28	09 30	10 02	09 20	11 30	3	32 E.	+ 10
Sept. 4	01 45	02 10	01 40	02 56	2	48 W.	+ 5
" 4	12 38	12 50	...	...	...	...	...
" 5	09 04	09 25	Sun under observation : see p. 607				...
" 10	12 16	12 31	...	...	...	...	...
Oct. 9	14 32½	14 48	14 54	15 03	3	40 W.	- 21½
" 21	15 34	15 48	Sun under observation : see p. 607				...
Nov. 6	16 13½	16 35	16 24	16 50	1.5	37 W.	- 10½
" 7	14 50	15 18	...	...	...	...	...
" 24	19 15	19 50	19 08	19 44	1	90 E.	+ 7
" 25	04 40	05 15	...	...	...	...	...
" 26	07 10	07 28	No scheduled observation				...
" 26	09 00½	09 28	...	...	...	...	...
" 26	17 55	18 18	{ 17 49	18 28	3	36 E.	+ 6
" 27	07 50	08 20	{ 18 40	18 43	2	38 E.	- 45
" 27	16 56	17 15	No scheduled observation				...
" 28	15 10	15 45	16 50	16 58	1	21 E.	+ 6
" 30	12 37½	13 08	...	...	...	...	...
Dec. 3	12 05	12 43	{ 11 46	12 18	2 +	55 W.	+ 19
" 9	13 12½	13 42	{ 12 17	12 45	3	56 W.	- 12
" 12	16 02	16 20	Sun under observation : see p. 607				...
" 19	08 45	10 00	Sun under observation : see p. 607				...
" 22	13 05	13 15	09 35	09 48	1	22 W.	- 50
" 23	09 50	10 15	...	...	...	...	...
" 25	08 55	09 10	...	...	...	...	...
" 26	09 13	09 35	Scheduled observation commences at 09 ^h 00 ^m				...
" 28	10 57½	11 10	...	...	...	...	...
" 29	08 21	08 41	Scheduled observation 08 ^h 00 ^m to 08 ^h 30 ^m				...
" 30	08 50½	09 10	(09 44	10 56	2	1 E.	- 53½) *
" 30	10 31	10 45	{ 09 44	10 56	2	1 E.	+ 47
" 30	11 00	11 20	{ 10 53	12 50	3	56 W.	- 22
" 30	11 00	11 20	{ 10 53	12 50	3	56 W.	+ 7
" 30	11 00	11 20	{ 11 00	11 05	1	48 E.	0
" 31	{ 13 00	14 12	Sun under observation : see p. 607				...
" 31	{ 13 50	14 15	13 45	14 20	2	12 W.	+ 5

* These two bright eruptions occurring just without the prescribed limit of time are entered here but are not included in the numerical data.

(3) In the last column is given the time-interval—Time of fading *minus* time of solar eruption, the figures again being in heavy type in those cases for which the origin of the solar eruption was observed.

An examination of Table II, containing data of 79 fadings, brings out the following points:—

(1) During 1935 (two years after the last solar minimum) no association of bright eruptions with radio fadings is suggested by the few occurrences tabulated. It will be shown later, however, that two or three significant coincidences were recorded from other radio data.

(2) During 1936, a year of rapidly increasing solar activity, no fewer than 29 close associations of radio fadings with bright solar eruptions are shown in the table. In 8 cases the fading was recorded before the bright eruption was picked up by the observer, but it is clear from the solar record that in 6 of these 8 cases the eruption had begun before the observer first located it. In 21 cases there was a bright eruption on the Sun actually in progress when the radio fading commenced abruptly.

(3) Taking those cases of association in which the origin of the solar eruption was observed, the time-interval *Radio Fading minus Observed Origin of Solar Eruption* is 7 minutes, derived from 12 occurrences.

(4) The 29 cases of observed association include a high percentage of the more intense solar eruptions.

(5) The distribution of solar eruptions associated with the 29 radio fadings shows no preference for the central part of the disk.

(6) The hourly distribution of the 79 radio fadings listed in Table II throughout the 24 hours U.T. shows a peak about noon, as given by the following tabulation:—

Hours U.T.	Number	Hours U.T.	Number
0-2	3	12-14	21
2-4	0	14-16	11
4-6	2	16-18	7
6-8	6	18-20	5
8-10	12	20-22	2
10-12	10	22-24	0

Since the emitting or receiving stations recording the radio fadings given in Table II are situated in terrestrial longitudes about which the Sun is on the meridian at about 12^h U.T., this distribution is in accordance with the characteristic already noted in paragraph 3, viz. that these radio fadings are observed, in general, on channels traversing a daylight path.*

Before stating the implication suggested by the above features, it is desirable to consider data of radio fadings derived from other sources. Whilst the present paper was being prepared, J. H. Dellinger published a paper on the same subject, entitled "Sudden Ionospheric Disturbances" (*Terrestrial Magnetism*, 42, 49, 1937). Included in Dr. Dellinger's paper is a useful table (*loc. cit.*, Table I) giving a list of occurrences of fadings (reported from at least two sources) during 1934-36 which he has collected

* This statement will be found exemplified by data circularized by Dr. Jouaust on 1936 October 15. See also *Journal B.A.A.*, 47, 100, 1937.

from various sources. From Dellinger's Table I we have compiled a supplementary table which contains 14 fadings during 1935 and 53 fadings during 1936 not listed in our Table II. (There are 31 fadings in our Table II not found in Dellinger's list.)

The diurnal distribution according to hours U.T. of these 67 fadings extracted from Dellinger's list is given by the dotted curve in fig. 2, the full curve being the distribution given by the European data in Table II. The data abstracted from Dellinger's list include a number of fadings recorded in the eastern half of the U.S.A., and he states that reports are relatively meagre from the Asiatic and Pacific regions. It will be seen in

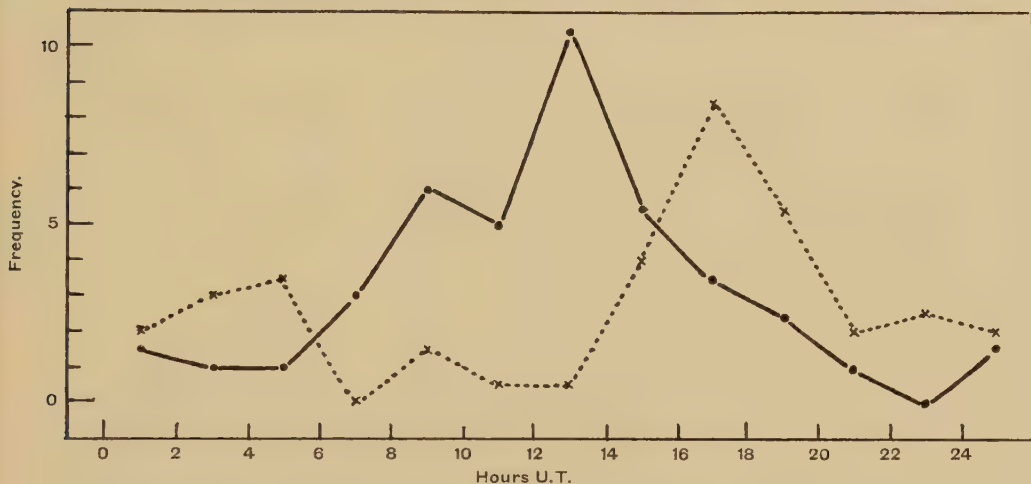


FIG. 2.—Diurnal distribution of radio fadings (commencements) according to hours U.T. (2 hourly means).

European data ————— Extra-European data (Dellinger)

fig. 2 that the peak shown in the diurnal distribution of fadings is displaced +4 hours from that given by data collected in our Table II.

The following table (Table IIA) is prepared in a similar way to Table II except that only joint occurrences (numbering 30 out of 67) of fadings and reported solar eruptions are listed. The durations of the fade-outs are also not available from the data to hand, but an average duration of 23 minutes derived from Table II has been used in making the comparison with the solar data.

5. *Discussion of Tables II and IIA.*—Combining the data in these two tables we have a total of 146 recorded radio fadings during 1935-36. In 59 cases there was recorded a bright solar eruption in close proximity, the mean interval—*beginning of fading minus time when solar eruption first seen*—being 4 minutes (the 20 best observed cases give 3 minutes).

This high degree of synchronization between the two phenomena may not, in itself, be significant, since owing to our prescribed limit of 30 minutes in associating a solar eruption with a radio fading, we should expect a small average time-interval to result. Taken, however, in conjunction with other features shown by the following analysis, the small time-interval of the order

TABLE IIA

(Supplementary to Table II)

Comparative Data of Radio Fadings (extra-European) and Bright Solar Eruptions, 1935-36, including only those Cases of a Joint Association (30 out of 67)

Date	Radio Fading (Commencement) U.T.	Bright Solar Eruption				Time-interval: Fading - Eruption
		From	To	Intensity	Distance from C.M.	
1935	h m	h m	h m		°	m
Aug. 30	23 20	23 12	23 30	2	3 E.	+ 8
Sept. 13	16 30	16 35	16 41	?	?	- 5
" 27	12 48	12 00	12 30	1	62 W.	+48
Nov. 29	14 05	14 31	14 45	1	33 E.	-26
" 30	18 55	17 51	18 30	2	19 E.	+64
Dec. 16	22 23	22 10	22 38	3	45 W.	+13
" 17	16 15	16 09	...	3	62 W.	+ 6
" 23	17 40	17 57	18 05	2	36 W.	-17
1936						
Feb. 16	15 50	16 30	17 00	1	25 W.	-40
Apr. 7	02 30	02 31	...	1	72 E.	- 1
" 8	09 20	08 10	10 00*	1	3 W.	+70
" 25	14 27	14 38	14 45	2	45 W.	-11
		16 15	16 30	2	41 W.	+38
		16 25	16 50	1	30 W.	+28
" 25	16 53	16 46	18 00	2	46 W.	+ 7
		16 49	18 00	1	35 W.	+ 4
May 30	17 30	15 42	18 00	1	44 E.	+108
June 9	15 55	15 30	17 00	1	80 W.	+25
" 9	17 50	17 55	18 05	2	77 W.	- 5
" 16	17 15	16 41	16 50	1	77 E.	+34
		18 01	18 09	2	70 E.	-46
" 16	18 00	18 01	18 09	2	70 E.	- 1
" 19	16 20	16 29	16 44	1.3	35 E.	- 9
" 19	17 30	17 36	18 00	2	28 E.	- 6
" 19	19 38	19 38	19 57	3	30 E.	0
" 19	20 18	19 38	19 57	3	30 E.	+40
Aug. 4	17 27	17 19	17 23	1	33 W.	+ 8
" 5	16 07	15 36	16 00	1	30 W.	+31
		16 03	16 48	2.3	63 W.	+ 4
Nov. 8	18 19	18 19	18 27	2	70 E.	0
" 16	15 00	15 30	17 00	1	73 W.	-30
" 25	19 25	18 40	19 04	1	50 E.	+45
		15 30	17 00	1	55 E.	+17
" 29	15 47	15 55	16 37	1	10 W.	- 8
		16 08	16 14	3	3 W.	-21
Dec. 24	22 05	22 00	22 18	1	72 E.	+ 5
		22 35	22 53	3	38 E.	-30
" 24	23 50	23 49	23 58	2	72 E.	- 1

* Three separate eruptions recorded between these times.

of a few minutes is probably of significance. It may be noted that a typical bright solar eruption takes from 5 to 10 minutes to reach its maximum intensity after the eruption first becomes visible (*The Observatory*, 59, 295, 1936).

These 59 associated eruptions have the following distribution in their recorded intensities as shown in Table III. (In those cases where two or more solar eruptions are associated in Tables II and IIA with a radio fading the more intense eruption has been selected.) In the last column the number of eruptions so associated in each class is expressed as a percentage of the total number of eruptions of that class observed during 1935-36 (see Table I).

TABLE III

Solar Eruptions associated with Radio Fadings, 1935-36, classified according to the Intensity of the Solar Eruption

Intensity	Percentage Frequency	Number of Eruptions Associated with Fadings	Percentage of its Class
1	77	22	4
2	19	23	15
3	4	14	48

The high incidence of the larger and brighter solar eruptions (which themselves are infrequent relative to those of intensity 1) found in association with radio fadings is very marked and suggests the reality of a relationship between the two phenomena.

We have now to consider that, on account of the relative frequency of solar eruptions (all classes taken together) during 1935-36, there is to be expected a number of chance coincidences with radio fadings, even if the two phenomena were entirely unrelated. The observed number of solar eruptions given in Table I probably requires correction by a factor of nearly 2, in order to allow for gaps in the international solar programme and for cloudy weather experienced frequently at some of the observing stations. The radio data, undisturbed by cloudy weather, probably requires a smaller correction on account of lack of continuity of observation, though at least three or four observing centres distributed in longitude around the Earth would appear to be required to ensure fully representative data.

In Table IV are tabulated the observed number (59) of radio fadings and associated solar eruptions during 1935-36 together with the expected number (n) of chance coincidences, computed by the following approximate formula:—

$$n = N \times f \times d,$$

where N = total number of radio fadings (22 in 1935 and 124 in 1936), f = mean daily frequency (corrected) of solar eruptions, d = mean duration of eruptions expressed as a fraction of a day.

Table IV brings out the fact that the observed number of coincidences between the two phenomena in question is far in excess of the number

TABLE IV

Coincidences of 59 Radio Fadings and Solar Eruptions

1935-36	(a) Observed Number.		(b) Expected Chance Coincidences	
	All Eruptions Intensity 1, 2 and 3	Major Eruptions Intensity 2 and 3	All Eruptions Intensity 1, 2 and 3	Major Eruptions Intensity 2 and 3
Year				
1935	8	5	0.4	0.06
1936	51	32	9.1	2.7

which would be expected if they were unrelated. This is especially marked in the cases of observed associations in which the solar eruptions were of major importance. It seems probable that a continuous solar record would considerably increase the number of observed coincidences.

Control Data.—In order to estimate further the extent to which chance coincidences may affect the result, two further tests on the data have been made. Firstly, we have taken the above 59 cases of association, and in each case have abstracted from the Zurich *Bulletin* the solar eruption and its intensity that was nearest in time to that already selected as occurring within the prescribed limit of 30 minutes. The average difference in time of these newly selected eruptions from the radio fadings is ± 7.6 hours, and the mean intensity of the eruptions 1.4, as compared with the corresponding values given by the original selection, namely, ± 0.1 hour : mean intensity 1.9.

Secondly, we have again taken the above 59 cases of associations and carried through afresh the comparison between the solar data and the radio fadings for the following epochs reckoned from 0^h = the time of each radio fading :—

$$-3^h, \quad +3^h, \quad -24^h, \quad +24^h, \quad -48^h, \quad +48^h.$$

The numbers of associated solar eruptions thus found within the prescribed limit of 30 minutes at each of these epochs were then tabulated, as before, according to their intensity, 1, 2 or 3. The results are given in Table V. In fig. 3 the numbers of associations found in each class of intensity are expressed as percentages of the total number of associations found in each class throughout the epochs, as defined above.

TABLE V

Number of Bright Solar Eruptions, according to their Intensity, occurring within 30 Minutes of Epochs 0^h , -3^h , $+3^h$, -24^h , $+24^h$, -48^h and $+48^h$, where 0^h = Time of Radio Fading

Solar Eruption of Intensity	Epoch from Radio Fading = 0^h						
	-48^h	-24^h	-3^h	0^h	$+3^h$	$+24^h$	$+48^h$
1	15	17	18	22	14	20	10
2	3	6	15	23	6	4	1
3	0	1	1	14	3	0	3

The results show clearly a peak at 0^h in the case of the larger and brighter

eruptions (intensities 2 and 3). [Incidentally, Table V suggests that within 2 days of the radio fadings there is a tendency for solar eruptions to rise above normal frequency.]

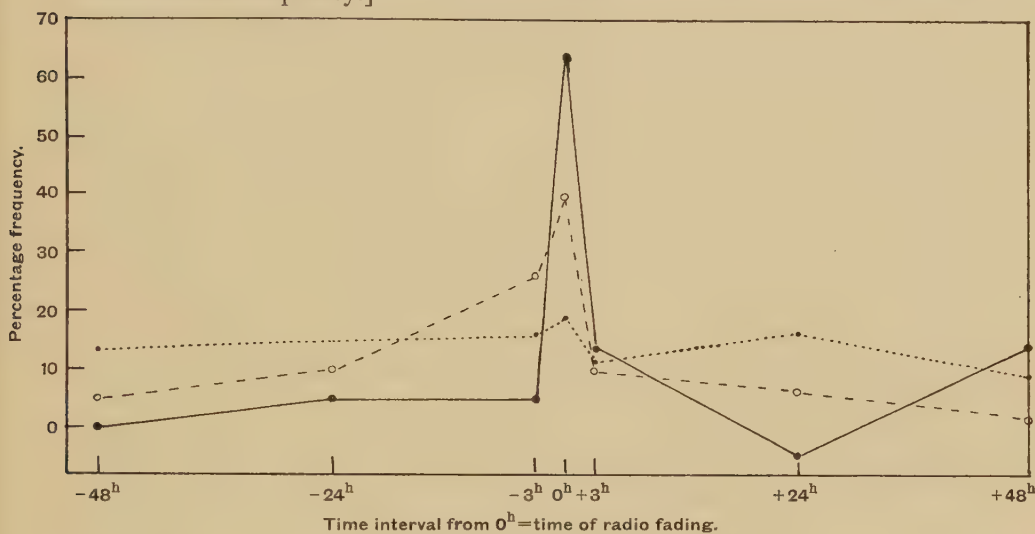


FIG. 3.—Solar eruption intensity

—— 3 (Greatest). - - - - 2. 1 (Least).

The distribution on the Sun's disk according to angular distance from the central meridian for the 59 solar eruptions associated with radio fadings may next be noted, as follows :—

TABLE VI

Longitude from Sun's Central Meridian	Number
0°-29°	14
30°-59°	27
60°-90°	17
Unclassified	1

Taking the evidence presented by Tables III-VI it may be concluded that there exists a close relationship between the observed radio fadings *

* It has been pointed out by R. Bureau and J. Maire (*Comptes rendus*, **203**, 1275, 1935; *Nature*, **139**, 110, 1937) that sudden fadings on short waves are often accompanied by improved reception on long waves (7500-11,000 metres). Their results are based on the automatic reception at Paris, Rabat and Tunis of atmospherics originating from a distant terrestrial source. In a memorandum circularized by the French Committee of Scientific Radiotelegraphy, there are listed, during the year 1936, 109 occurrences of a sudden increase in the number of atmospherics of which 47 occurrences (43 per cent.) correspond closely in time with the sudden fadings in short-wave transmission listed in Table II of our paper; 4 other occurrences correspond with fadings given in Table IIA. In 6 additional cases, not coincident with reported short-wave fadings, there were bright solar eruptions in progress. Although it is not proposed in this paper to extend the comparison of long-wave radio data and associated bright solar eruptions, it is noted that the former will provide useful and independent data for future discussion.

(implying disturbance in the ionosphere) and bright eruptions on the Sun. The observations indicate an agency travelling out radially in all directions, from localized patches of enhanced emission in the Sun's chromosphere, with approximately the speed of light. The more extensive bright solar eruptions appear to be most effective in this relationship.

It seems desirable to extend the astronomical observations in the following ways:—

(1) A more extensive record of bright solar eruptions, and any associated phenomena, in *H α* and in other absorption lines of the solar spectrum.

(2) A record to be kept at each contributing observatory of the times under which the Sun is actually under observation. This suggestion has been put into operation during the last few months, and the collective information assembled by M. d'Azambuja at the Meudon Observatory should eventually supply answers to the following questions.

(a) Can large solar eruptions observed in *H α* or in K occur on the Sun without affecting the ionosphere?

(b) Can appreciable disturbances in the ionosphere occur and no unusual disturbance be located with certainty on the Sun?

(3) An improved scale of estimating the intensity of solar eruptions, such as one based on photometry coupled with measurement of the area

TABLE VII

Solar Eruptions of Intensity 3 having (to date) no Reported Associated Radio Fadings, 1935-36.

Date	Time U.T.		Distance from Central Meridian	Observatory
	From	To		
	h m	h m	°	
1936				
Feb. 16	03 25	...	23 W.	K.
Mar. 16	16 24	...	8 E.	C.
May 15 *	07 04	08 30	27 E.	M. and Z.
June 2	16 31	18 00	45 W.	C.
" 5	15 40	16 26	22 W.	W. and G.
" 26	16 58	18 00	61 W.	C.
July 9	16 46	16 50	60 W.	C.
" 12	16 25	17 40	23 E.	C.
" 28	17 52	18 00	75 W.	C.
" 30 *	10 41	11 30	48 E.	G.
Aug. 3	05 03	...	2 W.	K.
" 5	22 17	22 21	49 W.	Mt. W.
" 15	07 46	08 20	1 W.	Z.
" 30	02 35	...	5 E.	K.
Sept. 25	04 18	...	5 E.	K.
" 26	02 23	...	0	K.

K. = Kodaikanal ; C. = (Wynne Cook) Swarthmore or Wynnewood ; M. = Meudon ; W. = (Newbegin) Worthing ; G. = Greenwich ; Mt. W. = Mount Wilson ; Z. = Zurich.

* On May 15 and July 30 Cable and Wireless, Ltd., report that conditions were somewhat erratic, particularly on the most northerly routes, as a result of magnetic disturbances.

of the enhanced emission. Whenever obtainable, an observed intensity curve of a bright eruption caught in its initial stages may be extrapolated backwards to derive the time of origin of the disturbance with greater precision than can otherwise be usually attained.

A table is appended giving a list of 16 large solar eruptions (intensity 3) observed during 1935-36 for which no associated radio fading has as yet been reported. During 1935 there were only two eruptions of intensity 3 observed; both of these were associated with radio fadings given in Table IIA.

It may be noted that the mean time difference of these 16 eruptions from Greenwich noon (12^h U.T.) is 6.0 hours.

The times of the 87 radio fadings unassociated with recorded bright solar eruptions have been compared with the Greenwich register of times of observations with the spectrohelioscope. In only 3 cases the Sun was being observed at Greenwich at the times of the respective fadings, viz. 1936 May 29^d 10^h 24^m, June 17^d 09^h 08^m * and Sept. 5^d 09^h 04^m. Although no bright eruption was registered, there was in each case appreciable or unusual activity observed at or near the time of fading, as follows:—

May 29.—The disappearance at some time between 11^h on May 29 and 10^h on May 30 of a long absorption marking extending over 30° of solar latitude.

June 17.—Two small patches of enhanced emission near a sunspot, not definite enough to be classified by the observer as a bright eruption; a measured component of radial velocity of +65 km./sec. of an associated absorption marking. Another velocity ranging from -30 to +50 km./sec. in an adjacent part of the disk.

September 5.—An absorption marking near a sunspot with a component of radial velocity of -130 km./sec. The marking projected about 1½' beyond the limb as a prominence.

M. d'Azambuja has kindly communicated the information that during the occurrence of four other fadings commencing 1936 Oct. 21^d 15^h 34^m, Dec. 9^d 13^h 12½^m, Dec. 12^d 16^h 02^m and Dec. 31^d 13^h 00^m respectively, the Sun was being observed at one of the collaborating stations. It is not known at present whether unusual solar activity occurred at or near these times, in the apparent absence of bright eruptions. A frequent record of the whole solar disk, taken with a self-recording spectroheliograph as recently installed at the Mount Wilson Observatory, would be of great use in these and similar cases.

6. Before summarising the results of this paper there are two points for inquiry that present themselves. (a) A tendency or absence of tendency for fadings to occur at intervals of 27 days, corresponding to the Sun's synodic rotation. (b) The association or otherwise of terrestrial magnetic phenomena at, or near, the time of solar eruptions accompanied by radio fadings.

* This fading is given in Dellinger's list in *Terrestrial Magnetism*, 42, 49, 1937.

(a) *Test for Recurrence Tendency of Radio Fadings in 27 or 54 Days*

The graph (fig. 4) has been derived in the following manner. Taking in turn each fading recorded either in Table II or IIA, an entry had been made on any day between the initial day = 0 and day = 60 when a fading is listed. The sums for each of the days 0 to 60 are then taken and expressed as percentages of the total for day 0 and plotted. The analysis has not been carried beyond 1936 November 1, since data for the requisite overlap into 1937 are not yet available. No extra weight has been given to those days on which two or more radio fadings are recorded.

Fig. 4 shows a very irregular curve * following the initial day of disturbance. Although a peak is shown at day = 28, there are similar peaks at days 6 and 36. There is also a broader peak centred at about day 52. The reality of these peaks appears doubtful, although, owing to the Sun's rotation in about 27 days, active sunspot regions producing bright eruptions and brought again on to the visible hemisphere by the Sun's rotation might be expected to produce a broad peak centred at 27 days (given a relationship between bright eruptions and radio fadings).

(b) *Associated Magnetic Phenomena*

Taking those cases of associated fadings and bright eruptions given in Table II, the magnetic traces of the Greenwich Magnetic Station at Abinger were examined for each of the occurrences. Out of 29 cases there were only 3 definite cases in which a small but possibly significant deflection of the H.F. trace occurred within a minute or two of the reported fading. The dates and times are 1936 April 8^d 16^h 49^m, May 28^d 07^h 29^m and Aug. 28^d 09^h 30^m. The ranges are quite small, of the order of 10 γ . On April 8 between 16^h 46^m and 16^h 51^m the horizontal force as registered at Huancayo (Lat. 12° 3' S., Long. 75° 20' W.) suddenly increased by 108 γ and then decreased 139 γ between 16^h 51^m and 17^h 28^m.†

Of the remaining 26 cases no deflections were recorded on the traces in 18 cases, while the remaining 8 cases were indeterminate owing to general unsteadiness or disturbance (including two small magnetic storms) that was prevailing at the time.

Next, the ranges in H.F. in 24 hours for the intervals, -48 hours, -24, +24, +48 . . . +120 hours, immediately preceding and following the time of the recorded beginning of each of these 29 cases were measured and tabulated in Table VIII. The mean ranges are given for

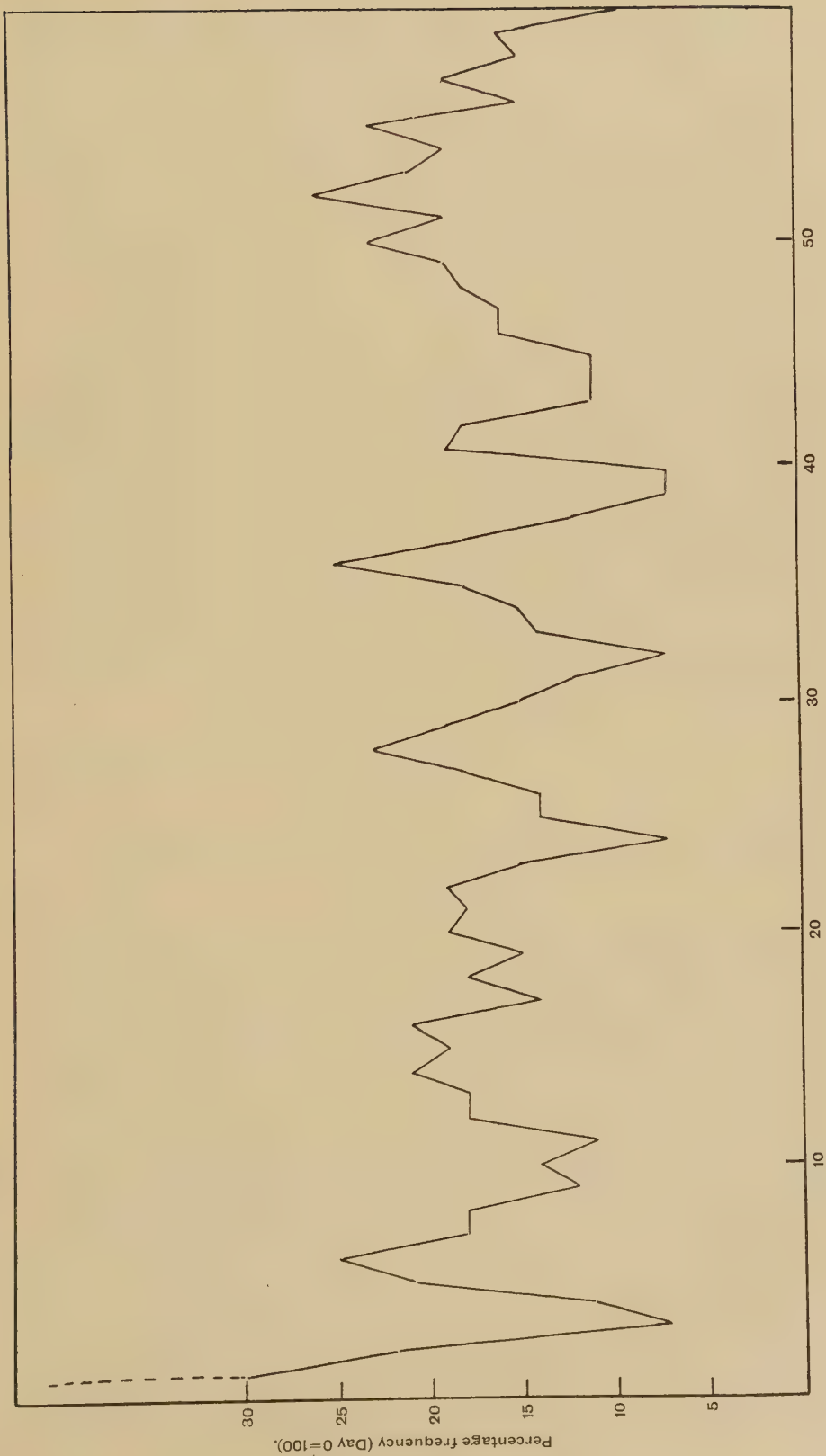
(a) All the 29 cases.

(b) Excluding the 2 cases when small magnetic storms were occurring.

(c) Eighteen cases (excluding one case when a magnetic storm was occurring) for which the associated solar eruption was within 4 days' time of the transit of the Sun's central meridian.

* This may be compared with the diagram given by Greaves and Newton in *M.N.*, 89, 642, 1929, which shows a 27-day period in the recurrence of the *lesser* magnetic storms.

† *Terrestrial Magnetism*, 41, 199, 1936.



Day from initial fading.

FIG. 4.

TABLE VIII

*Measured Ranges in H.F. (Abinger) Before and After Radio Fadings
associated with Solar Eruptions, 1935-36*

	-48 ^h	-24 ^h	+24 ^h	+48 ^h	+72 ^h	+96 ^h	+120 ^h
(a)	68 γ	71 γ	70 γ	71 γ	81 γ	62 γ	67 γ
(b)	70	62	66	62	67	59	60
(c)	67	61	63	68	75	61	63

The mean of the daily extreme ranges for the year 1936 is 68 γ .

It may be said that the above examination of magnetic conditions at Abinger at about the time of radio fadings observed from W. Europe during 1935-36 gives no definite indication that any associated disturbance took place in the Earth's magnetic field either at the time of the joint occurrence of solar eruption and radio fading or within 120 hours afterwards. The data, however, are insufficient to draw any definite conclusion. It is of interest to tabulate some similar magnetic data for the Earth as a whole given by the de Bilt magnetic character figures. The following tabulation shown in Table IX has been prepared by writing down the mean character figures for days -3, -2, -1, 0, +1, +2 . . . +6 for each of the radio fadings tabulated in Tables II and IIA for 1935 and 1936 September 30 up to which date the de Bilt circulars have been received.

TABLE IX

*Mean de Bilt Magnetic Character Figures corresponding to the Epochs of
37 Radio Fadings 1935-36 September 30*

	Day of radio fading = 0										
Day	- 3	- 2	- 1	0	+ 1	+ 2	+ 3	+ 4	+ 5	+ 6	
Mean of character figures	·61	·56	·69	·84	·71	·65	·64	·52	·67	·60	

In this case also no definite conclusion can be inferred from Table IX, but the trend of the figures suggests a slight rise in magnetic activity for the Earth as a whole on those days on which a radio fading occurred during 1935-36. There is no indication of a peak about two days after the radio fading.

It may be recalled that from the infrequent records since 1892 of bright solar eruptions of the greatest intensity there is shown a tendency for the eruption to be followed by a terrestrial magnetic storm about 26 hours later.* It is probable, however, that so far during the present 11-year cycle, no bright eruption equal in intensity to these very great eruptions has as yet been observed.

Attention has been drawn by various spectroscopic observers to the occasional occurrence of characteristic deflections shown in magnetic traces

* Hale, *Astrophysical Journal*, **73**, 379, 1931; (H. W. N.), *Monthly Notices*, **90**, 824, 1930.

synchronous with the observed time of great chromospheric outbursts at the Sun's limb.* Extended data based on the observations of bright solar eruptions and of disturbances in the ionosphere promise to throw light on these problems.

7. *Summary.*—The present paper makes a preliminary comparison between sudden radio fadings (implying disturbances in the ionosphere) and bright eruptions observed on the Sun in hydrogen light ($H\alpha$) or in that of ionized calcium (K). Certain characteristics of the radio fadings themselves suggest that they are related in some way to radiation effects proceeding from the Sun. The comparison made with the solar eruptions leaves little doubt that a close relationship exists between the two phenomena, especially in the case of the largest eruptions, and implies a solar agency, travelling from limited parts of the Sun's chromosphere (and indicated visually by bright $H\alpha$ eruptions) with approximately the speed of light.

The question of a tendency for radio fadings to recur in 27 days or 54 days is examined, as well as the possibility of terrestrial magnetic effects being shown when radio fadings occur. The results given by the present limited data are not conclusive enough to be stated with confidence.

It is obvious that the comparison between the bright solar eruptions and radio fadings should be continued during the present solar cycle. A future discussion should include definitive radio data as given by radio soundings of the ionosphere based on Professor Appleton's methods. A few suggestions are made in which the solar observations might be extended, but chiefly a continuous solar record, as far as weather conditions will allow, as well as an improved scale to define the intensity of the solar eruptions are required for furthering the present investigation.

In conclusion we wish to thank the Astronomer Royal for his suggestion that a comparison should be made between the solar and radio data put at our disposal.

We are also indebted to Mr. W. M. H. Greaves for some helpful criticism and advice when concluding this preliminary investigation, which in essentials follows the procedure which he used some years ago in a discussion † (with one of us) between sunspots and terrestrial magnetic storms. The latter relationship, as is well known, implies a corpuscular agency from the Sun.

Royal Observatory, Greenwich :
1937 June 1.

* C. A. Young, *The Sun*, p. 156, 1888 ; C. Michie Smith, *M.N.*, **70**, 23, 1909. The eruption referred to in this paper was on the disk but near the limb. A. Fowler, *The Observatory*, **56**, 43, 1933.

† *M.N.*, **88**, 556, 1928 ; **89**, 84, 1928 ; 641, 1929.

THE CONTOURS OF ABSORPTION AND EMISSION BANDS IN NOVA HERCULIS—FIRST PHASE.

E. G. Williams, M.A.

(Communicated by the Director, Solar Physics Observatory, Cambridge)

Introduction

The material discussed in this paper covers the period between 1934 December 13, the date of discovery of the nova, and 1934 December 21, the date of light maximum. This period presents the life-history of the

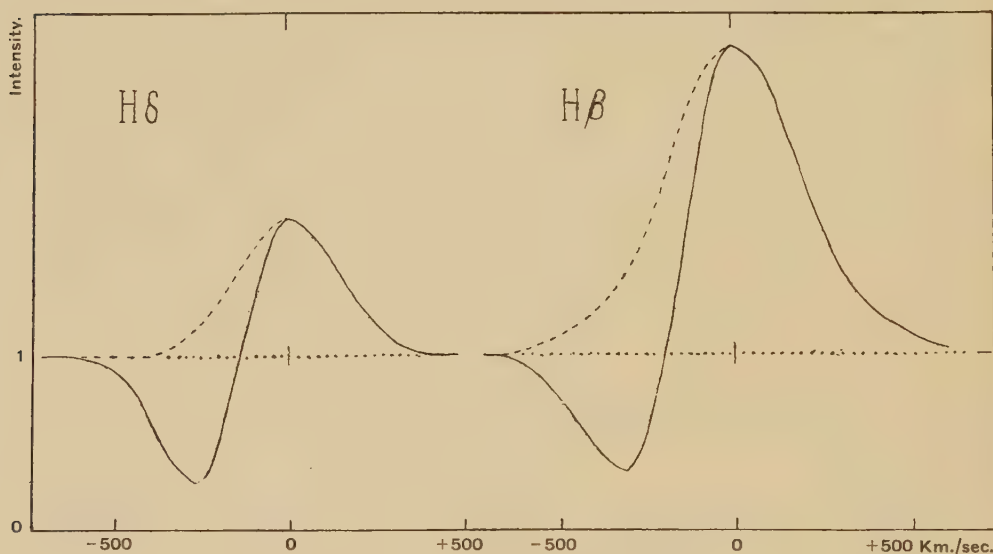
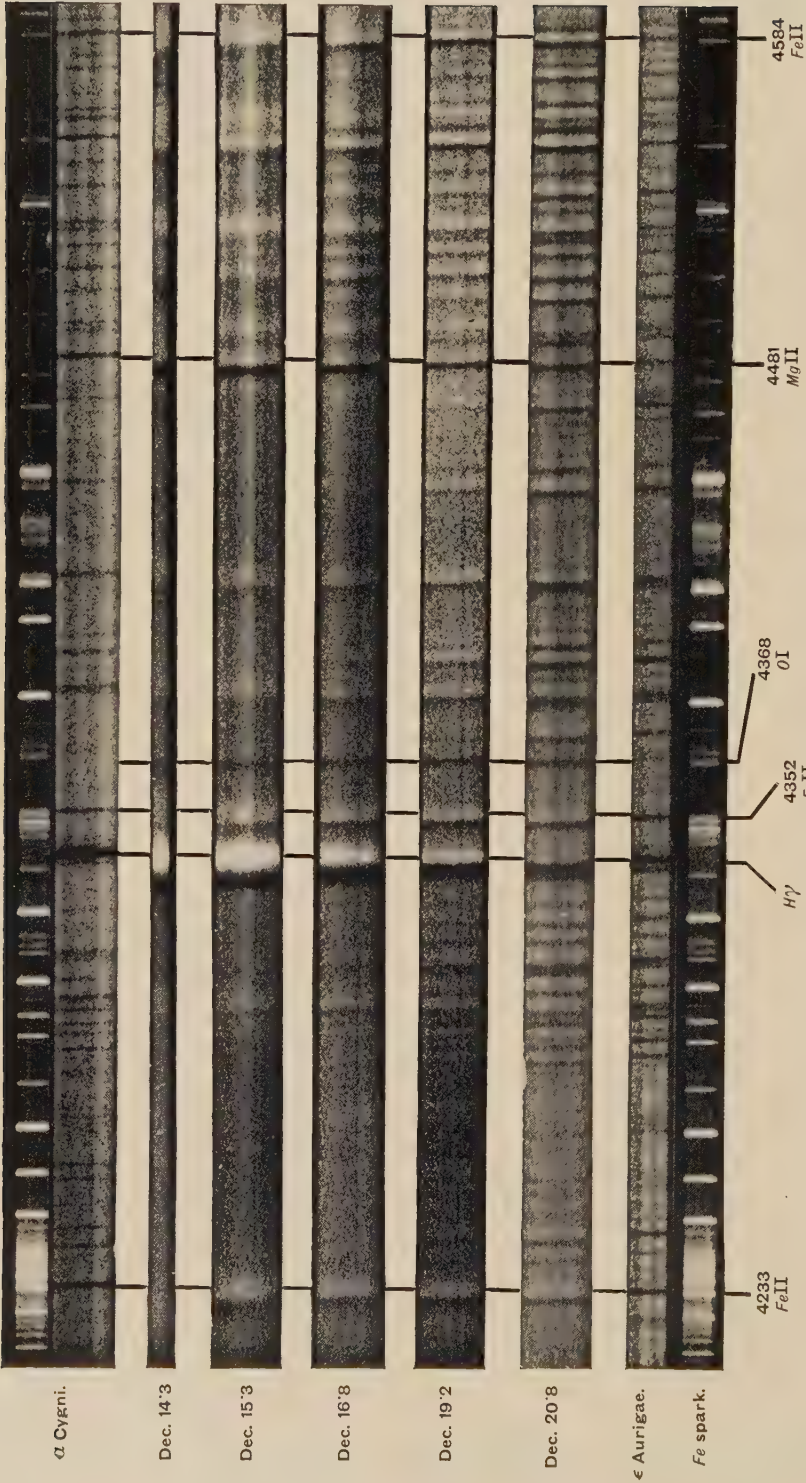


FIG. 1.—Mean contours of $H\delta$ and $H\beta$.

first absorption system, except for the last few days before its disappearance when no plates suitable for photometric work were obtained in Cambridge.

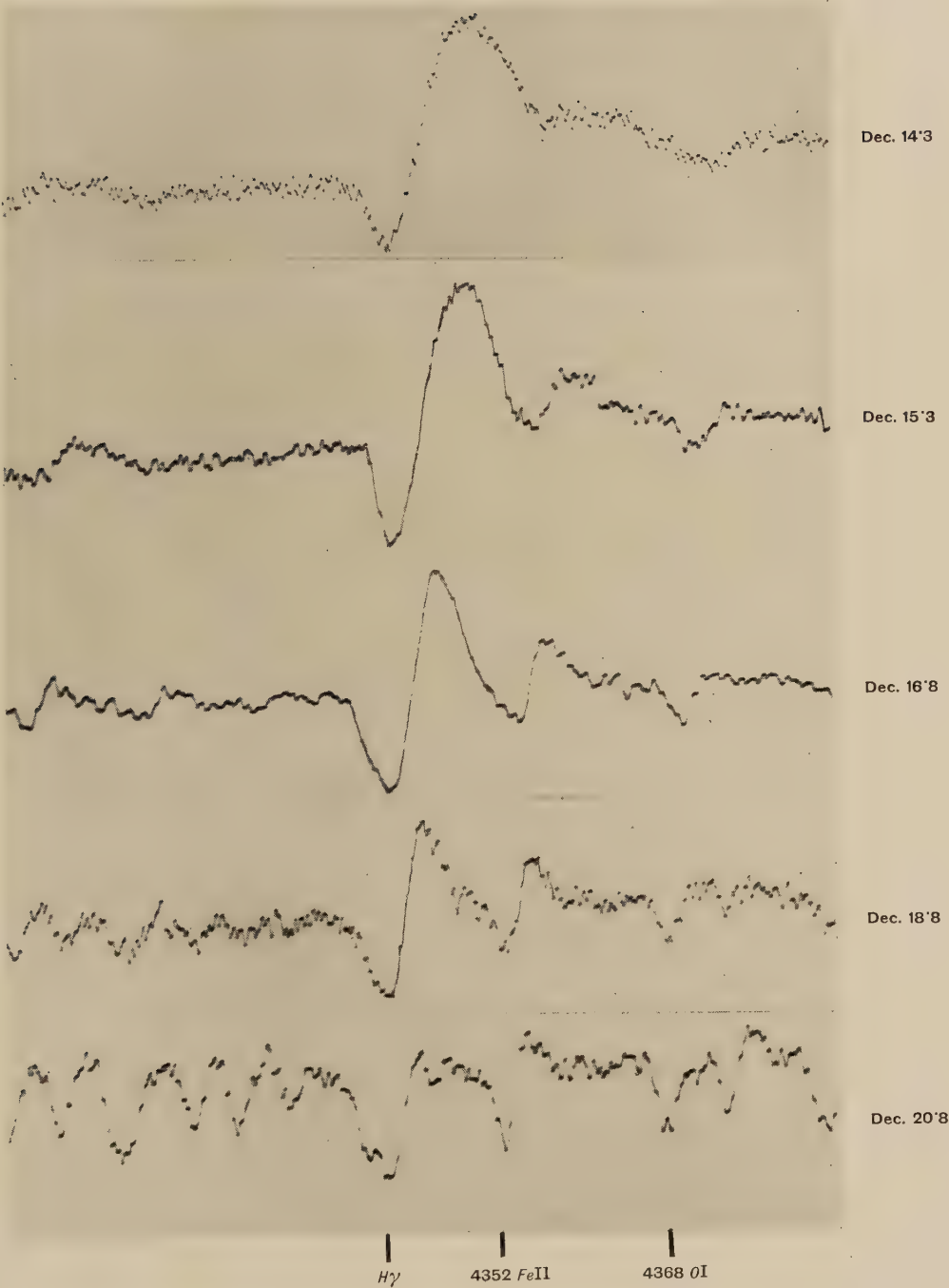
During this first phase the structure of the spectral lines was comparatively simple. Each absorption band was broad and single, much displaced to the violet, and flanked on its redward side by a strong broad peaked emission band. The maximum, or "peak," of this emission was almost undisplaced. No absorption lines appeared to the red of the emission; neither did emission appear on the violet side of the absorption. Thus the general appearance had much in common with that in the spectrum of P Cygni, though initially the displacements and band widths were much larger in the nova.

The structure is well illustrated by fig. 1, which shows the mean contours for $H\beta$ and $H\delta$ during the period under discussion. If the interpretation



Twelve-fold enlargements of spectra of Nova Herculis during the first phase. The increase in number and decrease in width of the lines is very marked.

E. G. Williams, Contours of Absorption and Emission Bands in Nova Herculis.



Microphotometer tracings of the $H\gamma$ region.

E. G. Williams Contours of Absorption and Emission Bands in Nova Herculis,

placed on the observations is that commonly adopted for novæ, we regard the emission bands as arising from matter surrounding the star in the form of an outward-moving atmosphere or of expanding shells of gas, while the absorption band is due only to those atoms of these shells which lie directly between the photosphere of the star and the observer, and are moving out radially with high velocity. The observed absorption and emission thus do not arise from identical parts of the outburst; consequently they should, in the first place, be treated separately. The measures which follow are presented and discussed on this basis.

The Spectra and Their Reduction

The discussion is based primarily on photometer tracings of sixteen calibrated plates taken with the Newall 25-inch refractor (see Plates 2 and 3). These cover the photographic region. In addition $H\alpha$ intensities have been obtained from four plates taken by Dr. Beer with the 15-inch Huggins refractor, while, through the courtesy of the Astronomer Royal, we have had the opportunity of studying the first three objective prism plates secured at Greenwich.

Of the Newall spectra the first is a 4-prism plate (scale 14 Å./mm. at $H\gamma$), and the remainder 2-prism plates (scale 39 Å./mm. at $H\beta$, 23 at $H\gamma$ and 15 at the K line). The 50-fold magnification was used on the recording photometer, so that the total length of record from λ 5018 to the K line is 7 feet. In all, over a hundred tracings were required. In order to reduce grain irregularities to a minimum the spectra were broadened on the plate to a width of 0.7 mm. whenever possible, though in a few cases the 0.3 mm. window was used. The plates were calibrated in an auxiliary spectrograph fitted with a Hilger wedge which was itself calibrated *in situ* for the wavelengths of $H\alpha$, λ 5500, $H\beta$, $H\gamma$, $H\delta$ and K. The usual precautions of employing backed plates, of brushing during development, and of calibrating with an exposure of comparable duration made simultaneously with the star exposure were observed.

For convenience of intercomparison all lines measured were plotted on intensity-velocity diagrams. The procedure was as follows: with the aid of the wedge calibration-curves, intensity marks expressing intensity in terms of the neighbouring continuous spectrum as unit were drawn across the features to be measured. Then the horizontal distances from a convenient reference line to each of these intensity-contour intersections were converted into kilometres per second and plotted on the velocity diagram so that the reduced shape could be sketched in with ease. Since the velocity displacement of both the absorption-line centre and emission maximum had previously been measured for these plates,* it was generally possible to fix the position of zero displacement on the velocity diagrams with fair accuracy. The measures given below are made with respect to this origin, which usually almost coincides with the peak of the emission.

* F. J. M. Stratton, *Ann. Solar Physics Obs.*, Cambridge, 4, 133, 1936.

The Measures

1. *Hydrogen Emission*.—The contours for the redward part of each line measured, expressed in terms of velocity displacements, are given in Table I.

TABLE I

Hydrogen Emission Contours Expressed in Velocity Displacements, with Peak Intensities, P , in Terms of the Neighbouring Continuous Spectrum

Intensities		Dec. 14 ^d .3	15 ^d .2	16 ^d .8	17 ^d .3	18 ^d .8	19 ^d .2	19 ^d .7	Mean of All
		<i>Hβ</i>							
<i>P</i>	...	3.36	2.90	1.85	1.75	1.56	1.21	1.05	...
	0.9 <i>P</i>	200 km.	130	130	90	95	110	40	110 km.
	0.7 <i>P</i>	330	230	180	150	170	170	90	185
	0.5 <i>P</i>	430	300	250	200	210	230	140	250
	0.3 <i>P</i>	500	400	330	250	260	290	230	323
	0.1 <i>P</i>	680	600	400	430	400	430	430	478
		<i>Hγ</i>							
<i>P</i>	...	1.59	1.83	1.12	0.99	0.80	0.69	0.53	...
	0.9 <i>P</i>	180	100	50	60	50	25	25	70
	0.7 <i>P</i>	280	180	110	100	110	75	75	132
	0.5 <i>P</i>	380	230	150	160	150	130	130	188
	0.3 <i>P</i>	480	250	210	230	230	190	190	253
	0.1 <i>P</i>	730	380	300	300	280	300	280	365
		<i>Hδ</i>							
<i>P</i>	...	...	1.27	...	0.72	0.62	0.52	...	...
	0.9 <i>P</i>	...	75	...	75	50	40	...	60
	0.7 <i>P</i>	...	150	...	130	100	100	...	120
	0.5 <i>P</i>	...	200	...	180	150	150	...	170
	0.3 <i>P</i>	...	280	...	230	210	200	...	228
	0.1 <i>P</i>	...	350	...	290	300	290	...	308

Here the unit of intensity is that of the observed maximum in the line. This peak intensity P , expressed in terms of the adjoining continuous spectrum, is given in bold type. The mean contours for the four comparable dates are shown in fig. 2, where the maxima are made to coincide. It will be observed that though $H\gamma$ and $H\delta$ agree excellently in shape the measured $H\beta$ is less peaked and throughout considerably broader.

It was surmised that this extra broadness of $H\beta$ might possibly be due to the limited dispersion of the spectrograph. The velocity scale on the plate at $H\beta$ is about 0.6 of that between $H\gamma$ and $H\delta$. In addition, the definition, at the settings used, was never quite as sharp at the longer wave-length. To test the matter the instrumental scatter curve in the $H\beta$ region was obtained through measures of iron comparison lines, and from it the change in shape of an idealized $H\beta$ was computed.

The result is illustrated by fig. 3, where the full curve represents an assumed line having the same width of redward emission as the observed $H\gamma$ but with intensity equal to $H\beta$, and the broken curve represents the instrumentally distorted shape. The scatter curve is also shown. This case is extreme in that the absorption on the violet side has been assumed intense enough completely to remove the continuous spectrum between the apparent line centre and the undisplaced position. It is clear that though the peak intensity is, in this case, decreased by 25 per cent. the observed extra width of $H\beta$ in the wings is by no means accounted for. The width

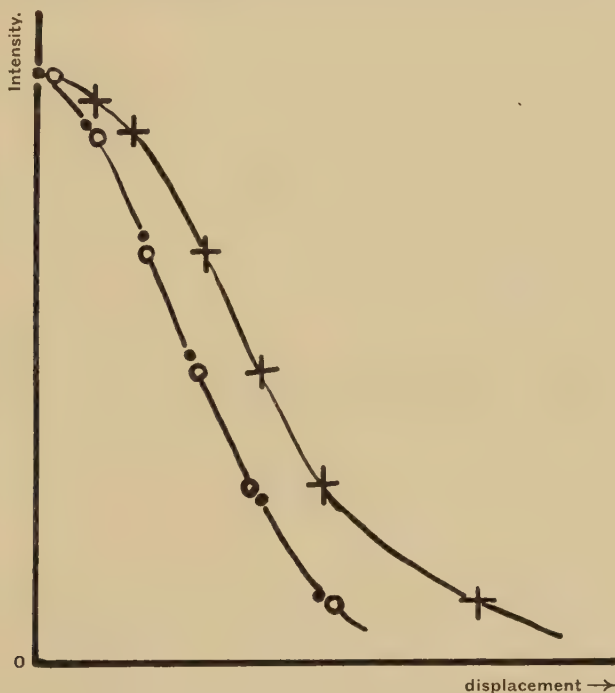


FIG. 2.—Mean contours of redward emission.

+ $H\beta$; O $H\gamma$; • $H\delta$.

ratio at intensity $P/2$ is *distorted line/assumed line* = 1.18, whereas the mean observed value $H\beta/H\delta$ is 1.40. In addition, since $H\gamma$ and $H\delta$ are also affected (though to a lesser extent) by instrumental redistribution, no account of which has here been taken, the *relative* distortion will be decidedly less than that shown in fig. 3. We are forced to conclude that $H\beta$ is in fact considerably wider than the other two lines.

In fig. 4 the widths also have been adjusted by a factor to coincide at half the peak intensity. On the left, points representing the contours are compared with a curve due to Gerasimovič * ($I = u^{-\gamma} - 1$, where u is, on his model, the velocity in terms of the escape velocity, and $\gamma = 0.426$ for best fit with our measures), and on the right with an error curve, which expresses the observations even more satisfactorily.

* *Zs. f. Ap.*, 10, 154, 1935.

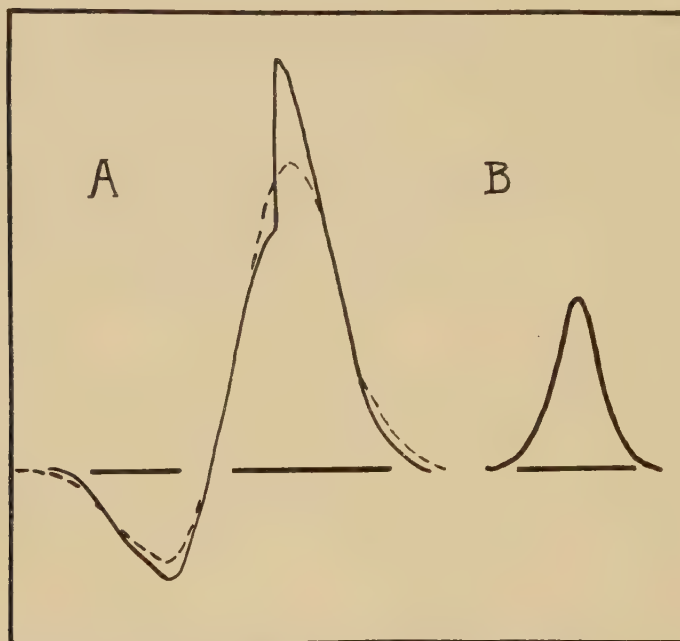


FIG. 3.—(A) Full line—assumed true contour ; broken line—instrumentally modified shape. (B) Scatter curve at $H\beta$.

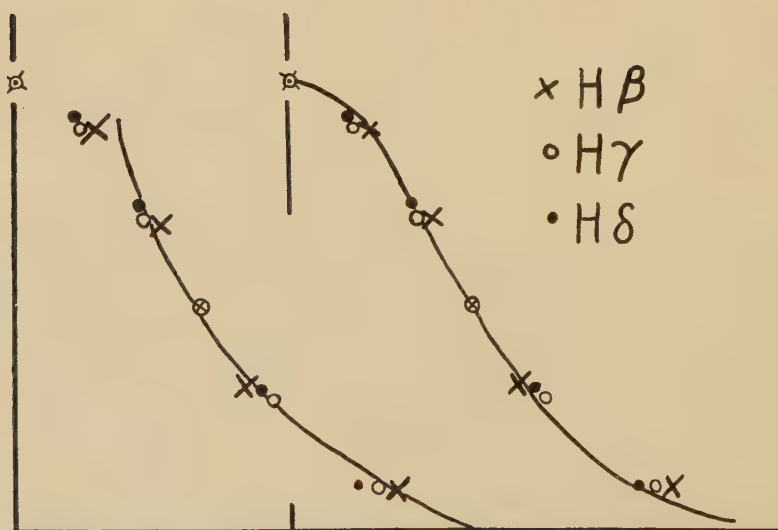


FIG. 4.—Mean shape compared with (left) curve due to Gerasimovič and (right) an error curve.

It should be mentioned, however, that at the beginning of the period under discussion the maxima of the lines were slightly more rounded than is shown by the mean contour, and at the end somewhat more peaked.

In Table II the integrated redward emissions between the contours of Table I and the continuous background have been doubled, to give total emissions, and expressed in equivalent angstroms of the neighbouring continuous spectrum. These intensities when converted into equivalent kilometres and divided by the respective peak intensities yield what we shall call the "effective emission width" of the line in question; the effective

TABLE II

*Total Emission Intensities and Effective Half-widths * for Hydrogen*

Line	Dec. 14 ^{d.3}	15 ^{d.2}	16 ^{d.8}	17 ^{d.3}	17 ^{d.8}	18 ^{d.8}	19 ^{d.2}	19 ^{d.7}	20 ^{d.8}	21 ^{d.3}	21 ^{d.8}
Total emission intensity (red + violet) in equivalent angstroms											
<i>Hα</i>	...	...	...	...	48.3	30.2	...	...	...	8.6	...
<i>Hβ</i>	48.4	33.8	16.1	13.0	...	11.7	9.71	6.27	3.00	1.7	3.09
<i>Hγ</i>	19.1	12.0	5.82	5.20	...	4.00	2.90	2.17	...	...	...
<i>Hδ</i>	...	7.8	...	3.45	...	2.80	1.92	...	...	...	0.23
Effective emission half-width in km. displacement *											
<i>Hβ</i>	443	359	268	229	...	231	249	184	232	231	265
<i>Hγ</i>	415	225	180	182	...	173	145	142	...	...	...
<i>Hδ</i>	...	225	...	175	...	162	135	...	...	...	106

* Effective width = ratio of total emission, in equivalent kilometres to peak intensity in terms of continuous spectrum as unit.

half-widths are given in the lower half of the table. They are useful for intercomparison of lines, since they give a measure of the spread of the band independent of the peak intensity, and illustrate the decrease in breadth of the bands which accompanied the diminution in absorption velocity. A similar rapid decrease in the width of the emission was recorded for RS Ophiuchi.*

The table includes also measures on three Huggins refractor plates of *H α* . The small scale in the red and the use of rather a wide slit precluded the possibility of deriving anything significant beyond the total intensity for this line. An approximate allowance was made for the absorption on the violet wing.

2. *Hydrogen Absorption*.—Measurement of the Balmer absorption is complicated by the existence of line emission as well as absorption on the violet side of the normal position. In what follows we have assumed that the emission is symmetrical about the undisplaced position, and have measured the absorption with respect to the reflected redward contour.

* Wilson and Williams, *Ap. J.*, 80, 344, 1934.

TABLE III

Hydrogen Absorption Line Contours Expressed in Velocity Displacements
(Violet wing of line only)

	14 ^{d.3}	15 ^{d.2}	16 ^{d.8}	17 ^{d.3}	18 ^{d.8}	19 ^{d.2}	19 ^{d.7}	20 ^{d.8}	21 ^{d.8}
<i>d</i>	<i>Hβ</i>								
0.0	...	...	...	...	650	580	580	...	575
0.1	...	...	625	620	560	530	500	...	465
0.3	730	625	540	570	487	440	432	380	325
0.6	650	570	473	480	400	362	350	302	265
Core *	545	458	330 :	328	270	245	238	230	180
C.I.	-6	-27	-28	+2	-10 :	-18	+7	0	+7
	<i>Hγ</i>								
0.0	...	740	690	620	620	575	480	blend	blend
0.1	...	625	590	530	470	470	437	...	...
0.3	680	570	520	450	375	375	367	...	...
0.6	600	505	392	362	315	305	270	...	233
Core *	500	435	318	278	207	225	210	...	175
C.I.	8	19	9	7	0 :	11	15	...	24
	<i>Hδ</i>								
0.0	...	725	...	600	490	630	...	...	545
0.1	...	660	...	463	380	438	...	...	392
0.3	...	600	...	375	308	350	...	...	318
0.6	...	530 :	...	300	280	272	...	...	245
Core *	...	420	...	245	208	200	...	...	182
C.I.	...	...	...	16	7	13	...	...	23

* Core = deepest part of the line, for which $1 - d$ varies from day to day.

Table III gives the extension of the violet wing for each absorption band, reckoned in kilometres displacement from the position of zero velocity, at five depths in the line. The measures have been made with respect to the hypothetical reflected emission wing at points for which the amount of absorption, d , from this wing is 0, 0.1, 0.3, 0.6. Here d (given in column 1) is the ratio of the intensity absorbed at the wave-length of measurement to the intensity of the neighbouring continuous spectrum. The displacement of the apparently deepest point in the line is also given, together with the value of $\frac{1-d}{100}$ for this point (*i.e.* the percentage central intensity). This is printed in italics. When it is negative the central absorption is greater than the total intensity in the star's continuous spectrum at that wave-length, so that some of the light from the overlying emission band is also being absorbed.

Fig. 5 reproduces the mean reduced contours of the violet absorption for the four days on which all three Balmer lines were measurable. The significant point about this figure is that, though the part of the contour

to the violet of the deepest part of the line moves in toward the undisplaced position as we pass from $H\beta$ to $H\delta$, the most displaced absorption visible, *i.e.* the extreme violet edge, remains in the same position for all three lines.* Between December 14 and 16 the displacement of the measured line centre

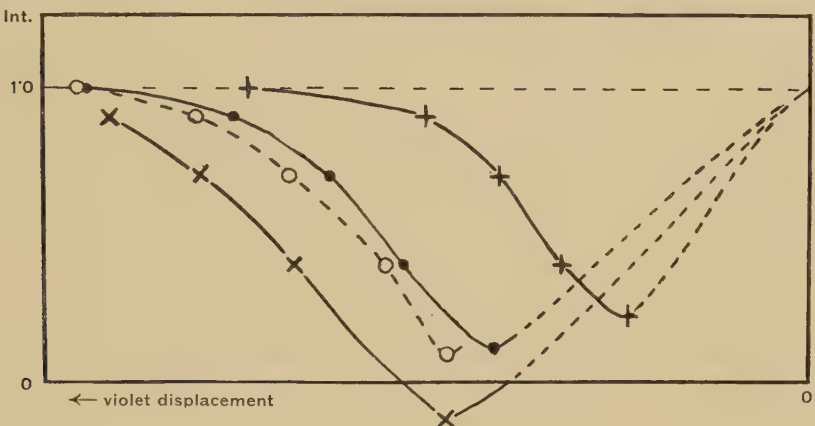


FIG. 5.—Mean band contours of the violet absorption.
× $H\beta$; ○ $H\gamma$; ● $H\delta$; + $\lambda 4233$ FeII.

The effect of the emission on the violet side of the undisplaced position has been removed.

was 55 per cent. of that of the extreme violet wing, but between December 18 and 21 it was only 37 per cent.

When the emission is weak it is not difficult to draw in the hypothetical emission wing over the violet absorption and then to measure the total amount of absorption included under this reflected line ; but if the emission is at all strong the deduced absorption becomes very sensitive both to the adopted emission contour and to the position assigned to the line centre. Hence the dotted sections of the reduced absorption curves in fig. 5 are relatively uncertain. In Table IV we have only measured the amount of

TABLE IV
Absorption-line Intensities Expressed in Equivalent Angstroms

Absorption to Violet of Line Centre	14 ^{d.3}	15 ^{d.2}	16 ^{d.8}	17 ^{d.3}	18 ^{d.8}	19 ^{d.2}	19 ^{d.7}	20 ^{d.8}	21 ^{d.8}
$H\beta$	2.65	3.45	2.22	2.75	2.70	2.80	2.30	3.10	2.24
$H\gamma$	2.56	1.46	2.06	1.77	2.06	1.66	1.59	...	...
$H\delta$	...	1.94	...	1.50	1.26	1.52	...	...	1.36
Reduced total absorption in $H\gamma$	4.29	4.34	3.62	3.66	3.25	3.27	2.55	3.24	2.57

* This is probably true also for $H\alpha$, since measurement of the extreme visible wing on a contrasty reproduction due to Merrill (*Ap. J.*, **82**, 413, 1935) gives, for December 21, a displacement just equal to that found here for the other lines.

absorption to the *violet* of the respective line centres. This area is comparatively well defined. The *total* absorptions have in each case been measured, but the uncertainty is such that it has been thought preferable to derive them from the tabulated violet absorption by applying smoothed values of the factor relating total to violet absorption.

The resulting figures for total absorption show that the relative equivalent widths are $\beta : \gamma : \delta = 1.00 : 0.82 : 0.68$ in kilometres, or $1.00 : 0.73 : 0.57$ in angstroms. Using these ratios the reduced mean *H γ* total intensity comes out as given in the last line of the table. These values are two to three times as great as those published * from Harvard objective-prism spectra covering the same period. Our figures indicate a steady decline from about $4\frac{1}{3}$ equivalent angstroms on December 14 to $2\frac{1}{2}$ on December 22. The latter value is closely equal to the intensity of *H γ* in α Cygni.

3. *Greenwich Spectra*.—Of the three plates examined only the first is calibrated. Nevertheless, in view of the importance of these early observations an attempt has been made here to derive order of magnitude intensities from the second and third plates by assuming similar characteristic-curves for the reduction. Though no great weight can be attached to the results of this procedure, they give us a useful picture of what was happening in the spectrum during the 24 hours following discovery. There is another cause of uncertainty. As the Greenwich spectra are slitless, it is impossible to fix the position of zero velocity on tracings made from them. Whilst it seems probable that the peak of the emission band marks this position for the ultra-violet lines and for *H δ* , it is doubtful if this is the case for the other lines. The absorption at *H β* and particularly at *H α* appears to be more intense than would be anticipated from the strength of the violet lines. Comparison of the positions of the apparent line centres on the plates shows strikingly that on the 13th (evening) the *H α* emission was heavily cut into by absorption, which shifted the apparent maximum in the line to the red of the position occupied on the mornings of both December 13 and December 14.

In deriving Table V, symmetry about the maximum has been assumed for lines below *H γ* , and absorption and emission areas obtained as for the slit spectra. For the three remaining lines the apparent emission above the continuous background is given in the first three columns. The true amount of absorption is here unmeasurable; an estimate of the total emission intensities in *H γ* has been made (in brackets) by extrapolating the absorption of the shorter wave-length lines and adding this to the observed emission.

In spite of the uncertainties necessarily involved, it is clear that there is a pronounced decrease in absorption intensity towards higher series members. The derived mean ratios (angstroms) are *H δ* 57, *H ϵ* 55, *H ζ* 38, *H η* 31, *H θ* 24. The threefold increase in relative emission intensity during the first day is also definite; this was caused by the striking increase in peak intensities, since the lines actually became less broad. The mean effective half-widths of the four lines *H δ* to *H η* are: December 13.3, 540 km.;

* Whipple and Gaposchkin, *Harvard Circ.*, No. 412, 1936.

TABLE V

Greenwich Objective Prism Plates. Hydrogen Emission and Absorption Expressed in Equivalent Angstroms

Line	Total Emission (Red + Violet)						Total Absorption		
	Observed			Corrected for Absorption					
	Dec. 13 ^d .28	13 ^d .75	14 ^d .3	13 ^d .28	13 ^d .75	14 ^d .3	13 ^d .28	13 ^d .75	14 ^d .3
<i>H</i> α	44	115	168	...	...	...	...	...	...
<i>H</i> β	14.6	32	42	...	...	...	...	...	...
<i>H</i> γ	4.3 *	14	16	(5.7)	(20)	(20)	...	...	...
<i>H</i> δ	...	...	10	3.55	15	13.7	3.55	5.4	4.1 :
<i>H</i> ε	...	...	...	3.18	12.0 †	12.3 †	2.32	4.15 †	5.97 †
<i>H</i> ζ	...	...	...	4.54 ‡	7.0	5.6	3.63 ‡	4.35	2.23
<i>H</i> η	...	...	...	2.24	3.8	5.7	1.73	2.86	2.56
<i>H</i> θ	...	...	...	2.22	...	4.4	1.46	...	1.76
<i>H</i> ι	...	...	...	1.51	...	...	...	...	...

* Blends with OII lines.

† Blends with CaII line.

‡ Blends with a strong *He* line.

December 13.8, 505 km.; December 14.3, 435 km. This last compares with the slit-spectrum value of 429 km. for *H*β and *H*γ at the same epoch.

4. *Balmer Decrement*.—The energy distribution in the continuous spectrum of the nova has been determined by the Greenwich observers for December 13, 18 and 23.* The peak intensities and total emissions of the preceding tables when multiplied by the appropriate values of I_{λ} , the energy in 1 A. of continuous spectrum, derived through linear interpolation between these dates, yield the true relative intensities for the lines.

If the line emission is considered as arising from atoms moving out radially from the nucleus, then those atoms and only those which are situated in the plane through the nucleus perpendicular to the line of sight will contribute the light in the undisplaced position of the line, and this light will escape through outer layers without absorption. Hence peak intensities of the hydrogen lines yield a Balmer decrement which is an *average* for all layers of emitting atoms in this normal plane. This is to be preferred to that from total emissions since *H*β is not the same shape as the other two measured lines. The figures are given in Table VI with the adopted nova magnitude for each date.

Excluding the last date, for which a real change seems to be indicated, the mean ratio *H*β : *H*γ : *H*δ is 1.0 : 0.76 : 0.57. When an approximate correction for differential resolving power is introduced this becomes 1.0 : 0.73 : 0.54.

* Greaves and Martin, *M.N.*, 96, 425, 1936.

TABLE VI

Peak Intensity in Hydrogen Lines in Terms of the Continuous Spectrum at $\lambda\ 5300$ in a 5th Magnitude A₀ Star

Line	Dec. 13 ^d .3	13 ^d .7	14 ^d .3	15 ^d .2	16 ^d .8	17 ^d .3	18 ^d .8	19 ^d .2	19 ^d .7	20 ^d .8	21 ^d .8
m_v	(3.2)	(3.4)	(3.1)	(3.4)	(3.0)	(3.0)	(2.6)	(2.5)	(2.5)	(2.1)	(1.8)
$H\beta$	(4.8)	(11.0)	24.2	15.6	14.3	13.5	17.2	14.5	12.5	6.8	8.8
$H\gamma$	...	(8.4)	15.3	13.6	11.9	10.7	12.4	11.6	8.7	...	3.5
$H\delta$	(2.4)	(7.5)	(9.5)	9.5	...	8.0	10.2	9.3	...	...	2.8

Mean values of total emission over three time intervals, considered in terms of $H\beta$ as unit, compare as follows :—

	$H\alpha$	$H\beta$	$H\gamma$	$H\delta$
Dec. 13.3–14.3 Greenwich plates	1.72	1.0	0.49	0.34
Dec. 14.3–16.8 Cambridge plates	...	1.0	0.51	0.32
Dec. 17.3–19.7 „	1.49	1.0	0.46	0.32

Since the $H\alpha$ measures are of low weight there is no appreciable evidence for change in the decrement between December 14 and 20.*

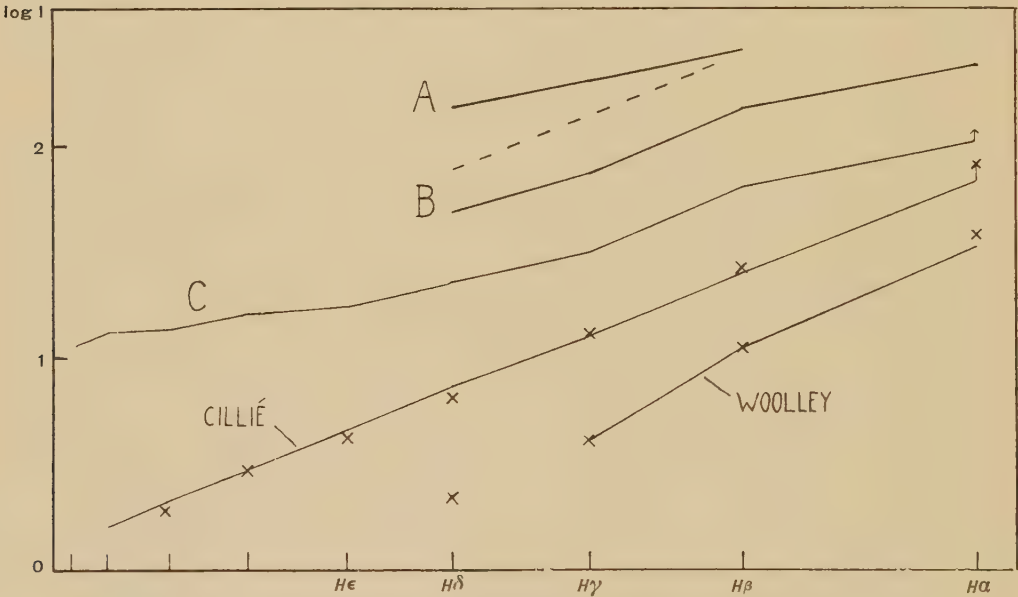


FIG. 6.—The Balmer decrement. A, from peak intensities ; B, from total emissions ; C, from Greenwich plates. The two lower curves represent theoretical values. The crosses are observed values ; those on Cillie's curve for planetary nebulae, and those on Woolley's for prominences. The broken line between A and B is parallel to Cillie's curve.

These results are illustrated in fig. 6. Here the vertical scale of intensities is logarithmic, while the horizontal scale is such that from an origin

* From the discovery plate Greaves and Martin (*Observatory*, 60, 65, 1937), measuring only the emission to the long wave-length side of the maximum, obtain $H\alpha/H\beta = 1.20$.

on the right (not on the diagram) the distances to the points marked α , β , etc., are proportional to $n^{\frac{1}{2}}$, where, starting with α , $n=1, 2$, etc. This method of plotting is convenient, since it leads to linear relations for the observed quantities. Curve A represents the peak intensity decrement and curve B that from total emissions, while C, to be regarded merely as an approximation, is derived from the three Greenwich plates on the assumption that the colour temperature in the region of the measured lines is the same as that in the visual region.

In the lower part of the figure theoretical values obtained by Cillié,* for electron capture ($T_e=10,000^\circ$), and by Woolley,† for polychromatic radiative equilibrium (Sun), are entered. The latest observational results for planetary nebulae‡ are indicated by crosses alongside Cillié's curve, and those for prominences on that representing Woolley's decrement.

5. *Metallic Lines*.—There are few of these which are both unblended and of sufficient strength to be worth measuring. Of those selected the *Fe*II lines $\lambda\lambda$ 4233, 4924, 5018 are effectively unblended; λ 4131 *Si*II and $\lambda\lambda$ 4179, 4352 *Fe*II are slightly blended in a way that can usually be allowed for; whilst λ 4481 *Mg*II and λ 4584 *Fe*II are badly interfered with on the red wing, although the violet absorption and peak intensity remain unaffected. Tables VII and VIII collect the measures. They are self-explanatory, being similar to those already given for hydrogen.

The relative absorption intensities of the iron lines compare as follows with those in α Cygni and in the Sun§ (intensities in equivalent angstroms):—

	λ 5018	λ 4924	λ 4584	λ 4352	λ 4233	λ 4179
Nova	4.72	3.68	1.84	1.14	2.10	1.00
α Cygni	...	0.80	0.78	0.50	0.62	0.52
Sun	0.161	0.131	0.105	blend	0.090	0.075

The ratio of strong to weak lines is much larger for the nova than in either comparison source. The relative behaviour in emission is less clear. Mean absolute values for the nova are here compared with Pannekoek and Minnaert's measures || and Mitchell's estimates ¶ for the chromosphere.

	λ 5018	λ 4924	λ 4584	λ 4352	λ 4233	λ 4179
Nova { Peak Intensities	1.75	1.9	1.8	1.4	1.85	1.0
{ Total Emission	3.5	3.3	3.1	1.5	3.0	1.0
Pannekoek and Minnaert	...	...	1.1	(0.8, blend)	1.7	1.0
Mitchell	25	30	25	(15, blend)	30	15

In fig. 7 the mean redward emission contours of the four unblended lines are compared with that for $H\beta$ on the same days. The curves are fitted at their maxima. The mean corrected absorption contour for λ 4233 was shown in the diagram for hydrogen (fig. 5), and illustrates well the lower displacements found in general for metallic lines during the first eight days.

* *M.N.*, 96, 771, 1936.

† *M.N.*, 96, 515, 1936.

‡ L. Berman, *M.N.*, 96, 890, 1936.

§ C. W. Allen, *Mem. Commonwealth Solar Obs., Canberra*, 1, No. 5, 1934.

|| *Verh. d. k. Akad. v. Wet. Amsterdam*, 13, No. 5, 1928.

¶ *Ap. J.*, 71, 1, 1930.

TABLE VII

Metallic Emission Lines

(Values in parentheses are from Greenwich plates)

$\lambda\lambda$	Dec. 13 ^d .7	14 ^d .3	15 ^d .2	16 ^d .8	17 ^d .3	18 ^d .8	19 ^d .2	19 ^d .7	20 ^d .8	21 ^d .8
Total emission (red + violet) in equivalent angstroms										
4131 <i>Si</i> II	(0.7)*	(1.0)*	—	—	0.7	0.4	0.4	—	—	—
4179 <i>Fe</i> II	—	—	—	—	1.4	—	1.3	—	0.2	0.3
4233 <i>Fe</i> II	(1.4)	4.8	5.0	2.8	2.8	2.8	2.2	2.4	1.5	2.1
4352 <i>Fe</i> II	—	5.1 :	2.6	1.9	1.9	1.8	1.7	1.0	0.5	0.3
4924 <i>Fe</i> II	—	13.5	10.6	5.2	5.6	5.9	3.9	4.1	2.2	2.1
5018 <i>Fe</i> II	—	11.7	11.4	7.0	8.3	7.7	5.2	—	—	2.9
Peak intensity in terms of the continuous spectrum at λ 5300 in a 5th magnitude A0 star										
4131 <i>Si</i> II	—	—	—	—	2.9	2.3	2.3	—	—	—
4179 <i>Fe</i> II	—	—	—	—	3.5	—	5.6	—	2.3	3.4
4233 <i>Fe</i> II	—	3.6	4.2	5.8	6.5	7.3	7.6	6.1	7.9	8.1
4352 <i>Fe</i> II	—	3.3	3.0	4.3	4.7	7.6	6.7	4.6	4.6	4.0
4481 <i>Mg</i> II	(2.5)	3.2	3.1	2.6	2.3	2.6	2.8	1.9	1.5	2.7
4584 <i>Fe</i> II	—	4.2	4.4	6.2 :	4.8	7.8	5.6	5.4	5.7	10.2 :
4924 <i>Fe</i> II	—	4.9	4.6	5.6	5.2	7.9	6.8	7.3	7.0	8.5
5018 <i>Fe</i> II	—	—	4.6	5.6	6.3	7.8	6.4 :	—	—	7.3
Effective half-width in km. displacement										
4131 <i>Si</i> II	—	—	—	—	92	107	115	—	—	—
4179 <i>Fe</i> II	—	—	—	—	153	—	140	—	80	82
4233 <i>Fe</i> II	(370)	460	312	185	167	219	177	236	161	274
4352 <i>Fe</i> II	—	520 :	225	156	152	129	150	118	80	79
4924 <i>Fe</i> II	—	570	360	214	245	240	200	194	153	163
5018 <i>Fe</i> II	—	—	352	255	270	286	260	—	—	209
Reduced <i>Fe</i> II Mean		492	266	187	194	202	185	183	129	161
Hydrogen Mean		429	270	224	195	189	176	163	—	186
<i>Fe</i> II Absn. Vel.		413	267	195	226	174	173	173	168	169

* The *Si*II pair blend here. Half-measured intensity given.

TABLE VIII

Total Absorption of Metallic Lines in Equivalent Angstroms

$\lambda\lambda$	Dec. 14 ^d .3	15 ^d .2	16 ^d .8	17 ^d .3	18 ^d .8	19 ^d .2	19 ^d .7	20 ^d .8	21 ^d .8
4131 <i>Si</i> II	—	—	—	0.50	0.39	0.33	—	—	—
4179 <i>Fe</i> II	—	—	—	0.75	—	0.75	—	0.99	0.96
4233 <i>Fe</i> II	2.15	3.13	2.17	2.20	2.15	1.86	2.12	1.50	1.68
4352 <i>Fe</i> II	—	—	1.00	1.24	1.36	0.99	0.81	1.02	0.77
4481 <i>Mg</i> II	2.75	2.90	2.18	1.82	1.72	1.39	1.10	1.02	1.10
4584 <i>Fe</i> II	1.83 :	2.27 :	1.90	1.91	2.30	1.65	1.57	1.47	1.59
4924 <i>Fe</i> II	5.50	5.58	2.97	4.56	4.09	3.10	2.97	2.50	2.16
5018 <i>Fe</i> II	—	6.46	—	5.90	5.75	2.45	—	—	—

The comparison at the foot of Table VII of the mean effective half-widths for iron and hydrogen also shows that the iron lines are, if anything, broader than the Balmer lines. For iron the half-widths are roughly equal to the absorption-line displacements, whereas in the case of hydrogen they are considerably smaller. Among the metallic lines there is a marked correlation between emission width and intensity.

Fig. 8 exhibits graphically the line-intensity variations during this first phase of the nova. It is seen that as the B-type spectrum dies out the

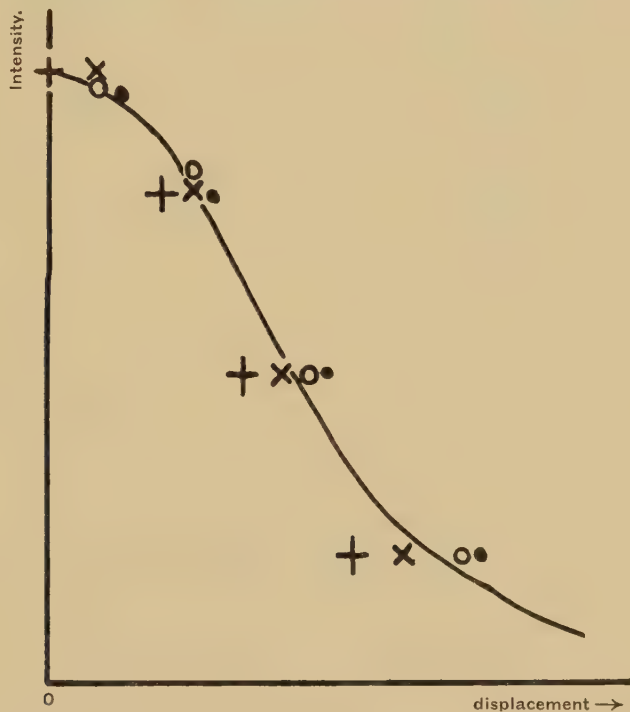


FIG. 7.—Contours of FeII lines compared with the mean $H\beta$ for same dates (curve).

$\times \lambda 4233$, $+\lambda 4352$, $\circ \lambda 4924$, $\bullet \lambda 5018$.

Balmer lines become much intensified just as they do in normal stars. After the hydrogen maximum, when the star is early A-type, the intensities steadily diminish till on December 21 the type is F2. The rapid change in relative strength of the hydrogen and iron emissions after December 19 is particularly noteworthy.

The changes shown in fig. 8 are normal for the high-luminosity branch of the stellar temperature sequence except for the behaviour of ionized carbon. The lines of this atom are altogether stronger with respect to helium than in ordinary stars, and persist until the A-type spectrum is well established. This observation is in line with others. Strong absorption lines of both neutral carbon and neutral oxygen are found throughout the first phase, though normally weak if present at all in stellar spectra, while the cyanogen bands appeared, December 24–29, and the strength of the forbidden OI

lines was striking from 1935 January onwards. This nova, and perhaps others,* therefore forms a link between the class R and R Coronæ Borealis type (class G with carbon bands)† and the carbon branch of the Wolf-Rayet group.

Utilizing the data of Tables VII and VIII, it is found that the strength of the emission relative to the absorption is greater for *SiII* than for any of the included *FeII* lines. The mean value of the emission-absorption ratio

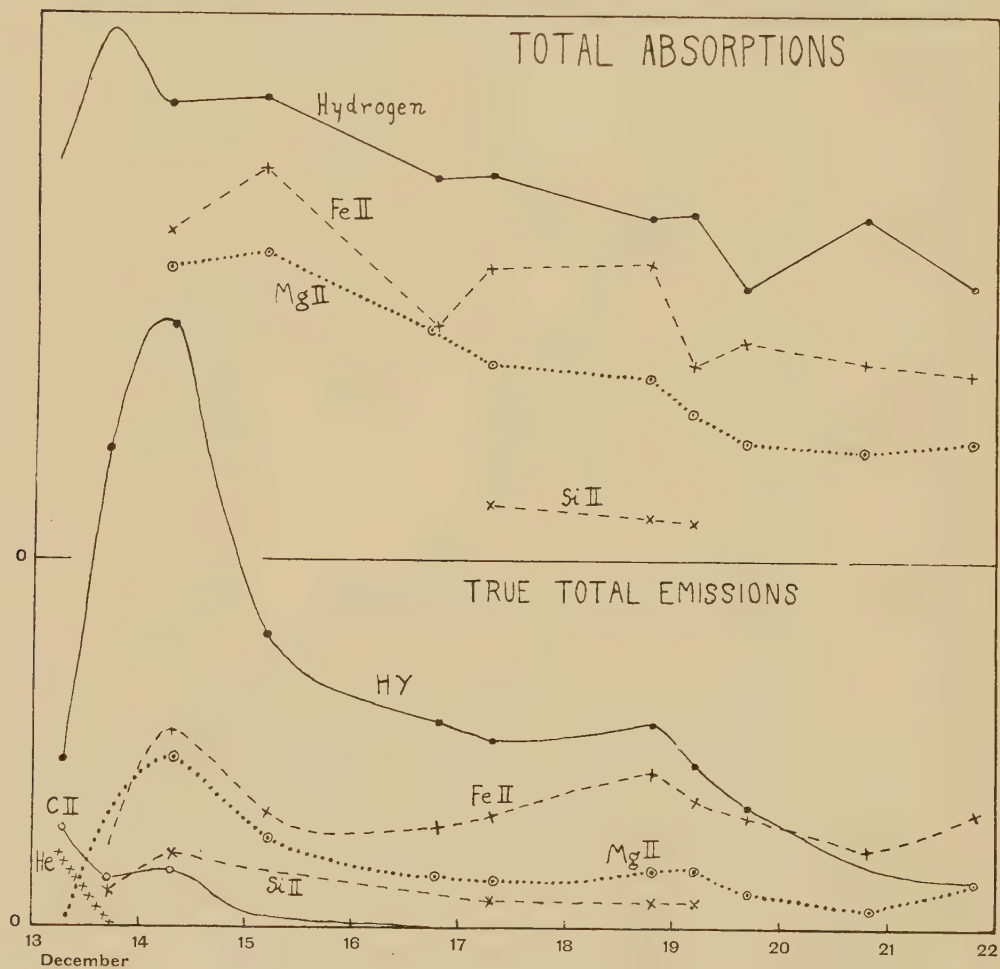


FIG. 8.—Variations in intensity of absorption and emission lines.

is larger for iron than for *MgII*, while among the iron lines themselves it decreases in the order $\lambda\lambda$ 4352, 4584, 4233, (4179), 4924, 5018, which is, except for λ 4179, the order of increasing intensity. In the case of hydrogen the ratio varies greatly from line to line. For the metallic elements it increases with time, roughly trebling itself in a week.

* McLaughlin (*Ap. J.*, 84, 104, 1936) finds that neutral oxygen and carbon lines are a common feature in novæ.

† C. P. Gaposchkin, *Harvard Bull.*, No. 903, 1936.

Although some absorption lines of neutral oxygen and carbon are quite strong the associated emissions are altogether weaker than the metallic lines would lead one to expect (see Plates 2 and 3).

Discussion

Any hypothesis of the nova outburst, to be wholly satisfactory, must cover, not only such salient features as (1) the presence of bright bands of considerable intensity, (2) their decline in absolute intensity during a period through which the visible photospheric radiation increased fivefold, (3) the concomitant decrease in displacement and width of each absorption line, (4) the change in spectral type from B to F without appreciable alteration in the colour temperature, and (5) the final fading-out of the associated absorption lines, but must also accord with the observed band contours and with the Balmer decrement. Indeed, analysis of the band contours suggests itself as the most hopeful line of attack in determining the mechanism of ejection, and in this connection the foregoing observations should prove helpful.

It is not immediately clear whether the extensive atmosphere surrounding the source of the continuous spectrum and giving rise to the line emission exists in the form of a discrete, radially expanding shell, or as a continuously ejected, outward-moving atmosphere. Similarity, during this first phase of the nova, to the band structure in P Cygni suggests the second alternative since the permanency of features in P Cygni rules out a single expanding shell.

Emission Contours.—The forms of possible emission contours on the hypothesis of continuous ejection have been investigated by several writers.* One family of curves derived by Gerasimovič accords relatively closely with the mean shape obtained in the present investigation† (see fig. 4). In deriving his contours, however, Gerasimovič neglected the occultation effect of the star on the redward wing (which wing of the line alone can be measured without interference from absorption), regarding this as of no importance for shells of gas *outside* ten photospheric radii. In the initial phases of the outburst and with the size of photosphere suggested by the colour temperature, it is found below that all of the emission originates *within* ten radii, so that Gerasimovič's treatment needs modification for application here. His assumption of no velocity spread in any one atmospheric layer also seems open to question in such an object as a nova immediately after its outburst.

When applied to the Balmer lines Gerasimovič ‡ finds that the adopted family of curves corresponds to decreasing outward velocity in the field of

* Beals, *Pub. Dom. Ap. Obs.*, **4**, 271, 1929; Gerasimovič, *Zs. f. Ap.*, **7**, 335, 1933; Chandrasekhar, *M.N.*, **94**, 522, 1934; O. C. Wilson, *Ap. J.*, **80**, 259, 1934.

† Except close to the line centre where the theoretical curve, $I \sim u^{-\gamma} - 1$, has a singularity.

‡ *Zs. f. Ap.*, **10**, 154, 1935. In this paper the same expression is found to apply well to hydrogen emission contours in N. Aquilæ, 1918, and in the repeating nova RS Ophiuchi.

the star at any one moment. Although identification of this model with the pre-maximum stage of Nova Herculis thus falls in line with the observed velocity decrease of the absorption lines, the association itself is of no great import unless the rapid deceleration can be accounted for. This deceleration presents the foremost difficulty attending the explanation of the mechanism of the first phase.

Stratton * and Minkowski † have both shown that the observed velocities imply inward accelerations which, when interpreted as gravitational, are too large to be reconciled with reasonable nuclear mass. As has been mentioned by these investigators, it is possible that the difficulty might be overcome by adopting a model for which the observed velocities refer to different atoms at different instants.

Hence, before accepting a hypothesis involving retardation, it seems worth considering the possibilities when the deceleration of ejected atoms is treated as illusory. We shall therefore suppose (a) continuous ejection of matter through a photosphere radiating as a black-body of temperature $11,000^\circ$. The magnitude and temperature observations combined with an estimate of the parallax ‡ then give a photospheric radius of $34\odot$ at the time of discovery, increasing at about 10 km./sec. until December 21. (b) The velocity of ejection, initially of the order of 800 km./sec. for hydrogen, decreases with time, attaining a constant value of about 170 km./sec. after nine days. (c) After ejection the velocity of any atom remains constant.

In order to reconcile assumption (c) with the apparent decrease in absorption velocity it is necessary to provide a means whereby the initially ejected atoms later become spectroscopically inactive. Three ways in which this might happen suggest themselves :

(1) Decrease through radial expansion in the number of absorbing atoms in a given atmospheric layer remaining in line of sight to the star. Though this would weaken the absorption due to the particular expanding layer, the corresponding emission would remain unaffected by the mere distance increase. Hence the observation that the emission width also decreases precludes an explanation in terms of thinning out of the gas.

(2) Disappearance of absorption through absence of exciting radiation. In the case of hydrogen, for instance, the outer atoms of the atmosphere might be completely shielded by the inner atoms from the photospheric radiation in the range of the Lyman continuous. The neutral atom on reaching the ground state must then remain there, and Balmer absorption is impossible. § The observations show, however, that the K line behaves similarly to the Balmer lines, and since K arises from the ground level of Ca^+ this explanation is untenable.

* *Observatory*, 59, 325, 1936.

† *Ap. J.*, 85, 18, 1937.

‡ E. G. Williams, *M.N.*, 95, 573, 1935.

§ Whipple and Mrs. Gaposchkin point out (*Proc. Nat. Acad. Sci.*, 22, 195, 1936) that the excess Lyman α radiated from the lower parts of the atmosphere can keep the hydrogen atoms of the upper layers in a state to absorb the Balmer series. If, however, there is an outward velocity increase as supposed here, the outer atmosphere is transparent to $L\alpha$ radiated from the inner atmosphere.

(3) Loss of absorbing power through ionization or second ionization as the pressure decreases with expansion. This seems to be the most hopeful solution and may apply equally to the disappearance of rising prominences. The acceptance of this mechanism may, however, involve us in difficulties if an attempt is made to identify some of the absorption components, visible at a later stage of this nova, with expanding shells, since these components remain over a prolonged period.

In attempting to get an idea of the form of emission contour given by this model, the expanding atmosphere will be treated as though consisting of seven separate layers within each of which all atoms are moving outwards with the same velocity. We can then apply to each of these layers Beals's original result,* which shows that the contour due to a single spherical expanding shell is rectangular in shape, symmetrical about the position of zero radial velocity, and has its flat top extending to the positive and negative displacements corresponding to the particular outward velocity of the atoms in the shell.

The assumed data are given in Table IX. The velocities are not those of the absorption "lines," but are proportional to the widths of the emission bands on each date.

TABLE IX

Layer	1	2	3	4	5	6	7
Date of ejection	Dec. (12.3)	13.3	14.3	15.3	16.3	17.3	18.3
Velocity (km./sec.)	(900)	600	450	290	230	210	200
Radius of layer on Dec. 18.8 (astronomical units)	(3.2)	2.3	1.54	0.93	0.71	0.56	0.45
Resolved velocities oc- culted	(895)	590	435	262	190	150	93

Fig. 9 is a representation of the state of the outburst on December 18.8. Here the central circle is the photosphere with radius $43\odot$, and the others show the mean position of atoms in each of the layers at this date. The freshly ejected atoms are moving with less than a quarter the velocity of the outermost atoms.

Fig. 10 illustrates the built-up contour on the assumption that each layer (which consists of the atoms ejected during a 24-hour period) contributes the same total emission intensity, *i.e.* the superposed rectangles are equal in area. The band is flat-topped over a range of ± 200 km./sec. Next, the occultation on each layer by the star must be taken into account. For each rectangle (corresponding to a concentric circle in fig. 9) there is a definite displacement to the redward side of which all light is cut off. With the assumed photospheric size these displacements are as tabulated in the fifth line of Table IX.

* *Pub. Dom. Ap. Obs.*, 6, 95, 1934.

In fig. 10 a heavy curve has been drawn through the positions of cut-off for the various layers, and shows the smoothed shape of the remaining redward emission.

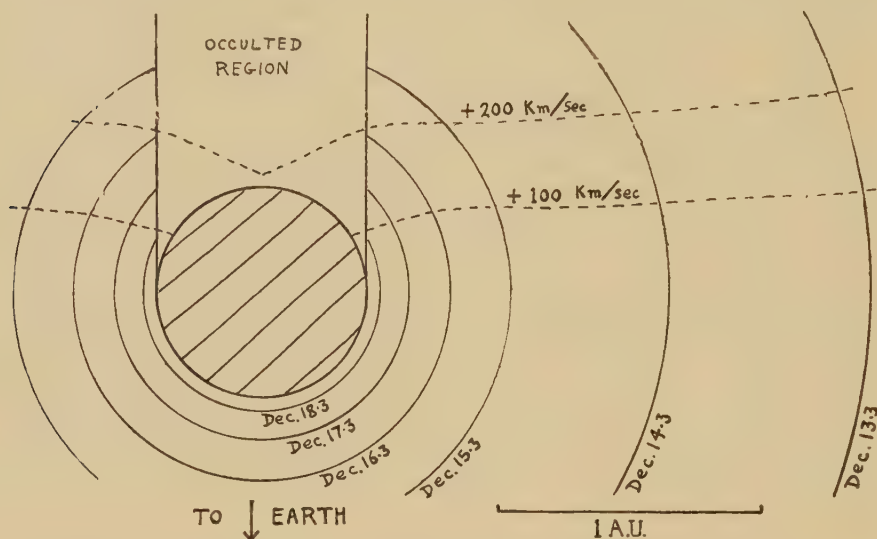


FIG. 9.—The positions of expanding atmospheric layers on December 18.8. The broken lines connect points at which the resolved velocity towards the earth is constant.

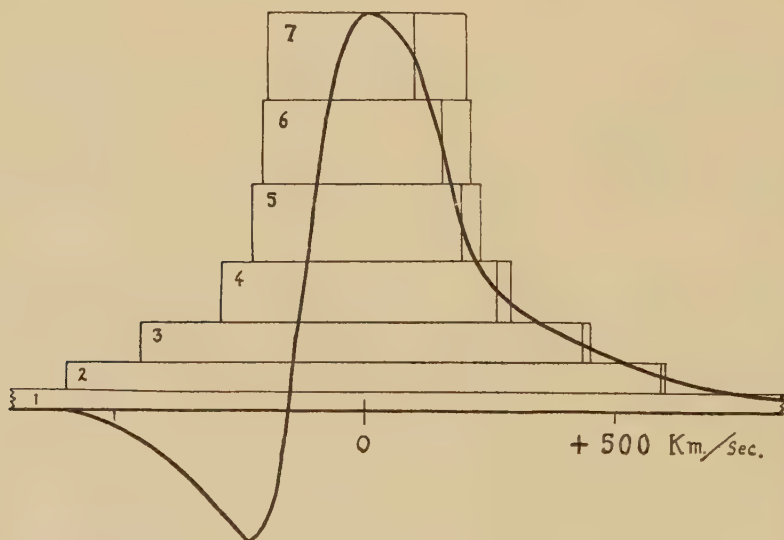


FIG. 10.—Built up emission band based on a model involving constant velocity of ejected particles coupled with decrease of ejection velocity. Occultation by the central star on the atmosphere is allowed for. The numbers refer to the layers in Table IX.

On the violet side the emission overlies the absorption band in the continuous spectrum; the amount by which the emission appears to be cut away depends, therefore, on the relative intensity of the underlying

continuous. The heavy line on the violet side in fig. 10, representing the observed contour, is merely illustrative.

The built-up shape of the redward emission is quite similar in character to the observed contour in Nova Herculis. Actually, the assumption made, that the emission from all layers is equal, will not hold. If the intensity decreases rapidly to vanishing-point through the three outer layers the wing of the curve becomes curtailed, and the agreement with the observed shape (fig. 1) is even closer.

In addition to their contours the strengths of the emission bands need explanation. The difficulty here is that a black-body at $11,000^\circ$ (corresponding to the colour temperature of Nova Herculis) does not emit enough quanta beyond the Lyman limit to give the observed intensities on recombination. If, however, sufficient atoms could be kept in state 2 the photospheric radiation at the Balmer limit (which is, at this temperature, 43 times as intense) would also become available for ionizations. This is not easy, though strong Lyman α , the metastability of the $2s$ level, and collisional ionizations through turbulence would assist.

As was shown in fig. 6, the Balmer decrement is decidedly slow, the ratio $H\alpha : H\gamma$ being about 3. Woolley's calculations * for polychromatic radiative equilibrium, appropriate to the case of prominences, do not differ very greatly from those given by Cillié † for electron capture. (The ratios $H\alpha : H\gamma$ are Woolley 8.3, Cillié 6.0.) Cillié's result applies to nebulae, where nearly all hydrogen atoms are either in the ground state or ionized, while Woolley's is for a region of much higher radiation density; presumably the nova case is intermediate. The degree of departure between theory and observation is illustrated by fig. 11. The upper curve represents the contour of $H\beta$ deduced from the observed contour of $H\delta$, assuming similarity of intensity distribution and also that the relative intensities are as computed for electron capture. The lower curve is the observed $H\beta$. Woolley ‡ suggests that self-reversal is responsible for the discrepancy.

Actually, as can be seen from fig. 11, $H\beta$ is found to be broader than the other lines, § so that the decrement is steepest (steeper, indeed, than required for electron capture) in the most displaced part of the line, which we identify with the outer atmosphere. This is evidence that the stellar radiation is weak enough to be absorbed completely before the outer layers are reached, and that, in the latter, emission follows line absorption.

A theory in which the nova could produce many more high-energy quanta than would be given by a black-body at the observed colour temperature has been put forward by Whipple and Mrs. Gaposchkin. || Their

* *M.N.*, 96, 515, 1936.

† *M.N.*, 96, 771, 1936.

‡ *Loc. cit.*, p. 522.

§ Though our spectra suggest no escape from this anomaly a check observation would be welcome. The very close agreement in shape of $H\gamma$ and $H\delta$ throws additional suspicion on $H\beta$, though it is possible that the extreme wing of $H\gamma$ is curtailed by $\lambda 4352$ absorption, in which case there would be a slight progressive change in character.

|| *Harvard Circ.*, No. 413, 1936.

discussion of the continuous spectrum emitted by a very extended expanding atmosphere yields in the outer layers a large excess of radiation in the region of the Lyman limit as compared with the Planckian energy distribution. This result might account for the appearance at discovery of B-type lines when the visual colour-temperature was only $10,800^{\circ}$.^{*} At later dates during the first phase, however, reasonable correspondence existed between colour and spectral type,[†] so that, if such a copious supply of high-energy quanta is to be provided, some loophole must be left to account for the absence of the high excitation lines.

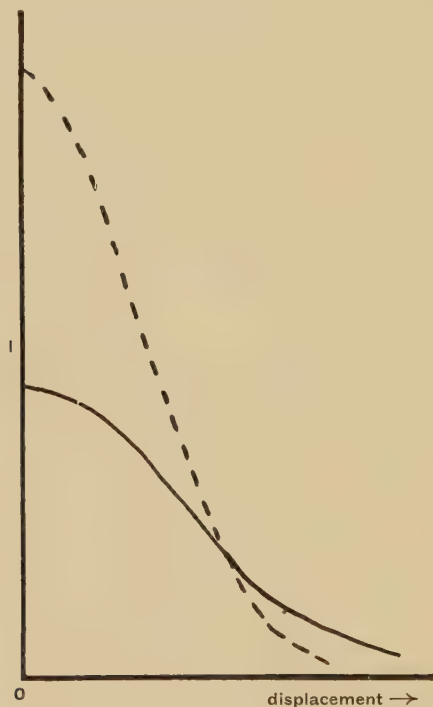


FIG. 11.—Comparison of observed $H\beta$ (full line) with contour obtained from $H\delta$ by multiplying ordinates by 3.45.

Absorption Contours.—The form of an absorption line due to an expanding atmosphere has been investigated by O. C. Wilson.[‡] His result refers to that part of the atmosphere close to the photospheric surface, so that some of the absorbing atoms are moving at a large angle to the line of sight. He finds that for a line of small intrinsic width and for a nova-like velocity the observed absorption extends from the undisplaced position to just beyond the displacement corresponding to the ejection velocity, whilst the line is very asymmetrical, its deepest part being well out towards the violet wing.

^{*} Greaves and Martin, *Observatory*, 60, 65, 1937.

[†] J. S. Hall (*A.J.*, 45, 197, 1936), studying the colour with filters transmitting at 8000 Å. and 6300 Å., finds colour-class A5 on December 15.7 and F2 on December 20.7.

[‡] *Ap. J.*, 82, 233, 1935.

For gas far from the star, on the other hand, the line becomes quite sharp. If there is an outward velocity increase (such as is here assumed) the line broadens into a band whose intensity at a given displacement depends on the number of atoms moving with the appropriate velocity in the line of sight to the star which are excited to the lower state of the line in question.

The shape of the absorption lines in Nova Herculis as measured under a reflected emission contour is very roughly triangular, and the inner edge coincides with the position of zero velocity * (fig. 5). When the occultation effect on the redward wing is taken into account this shape requires modification and the total absorption is somewhat increased.

The interpretation placed on the absorption contour is that the least displaced parts of the line refer to atoms close in to the star moving at large angles to the line of sight, and that the violet wing arises from the outermost atoms which are able to absorb from the Balmer level. In the violet wing saturation cannot occur or the line would be black. The increasing intensity here in passing from $H\delta$ to $H\beta$ is puzzling.

It has been noted for $H\beta$ (Table III) that the line centres are on occasion so deep that there must be absorption not only of the whole of the underlying continuous spectrum, but also of part of the emission on the violet side.† This indicates, as Whipple and Mrs Gaposchkin have pointed out,‡ that the atoms giving rise to the absorption are exterior to those causing the emission and do not themselves contribute appreciable emission. This would be natural enough in our model were it not that the geometry of the problem requires that an outer atom in the line of sight to an inner atom must have a *smaller* outward radial velocity if it is to be able to absorb the line emission of the latter.§|| The difficulty may, perhaps, be met if large velocity spread is allowed in the absorbing layers. Such turbulence is to be expected in the early stages of a nova outburst.

Metallic Lines.—During the first eight days following outburst the velocity determined from the metallic absorption lines was markedly lower than that from hydrogen. The spectrophotometric measures show, however, that if the emission widths are taken as velocity criteria, both the wider contours and greater mean effective widths found for the $FeII$ lines (Table VII) point, from the first, to slightly *greater* outward velocities for the heavier atoms.

The Cambridge measures ¶ on the two strong $FeII$ absorption lines $\lambda\lambda$ 4924, 5018 give, between December 15 and December 18, values about 30 per cent. in excess of the mean iron velocity. This, taken in conjunction

* This observation is strong evidence against the simple shell hypothesis, since a shell detached from the photosphere and expanding with large velocity would give no absorption close to the undisplaced position.

† We assume an explanation invoking unsymmetrical emission to be improbable.

‡ *Observatory*, 60, 46, 1937.

§ Rosseland, *Theoretical Astrophysics*, p. 332. Oxford, 1936.

|| Gerasimovič's scheme in which, at any instant, the velocity decreases outwards according to the relation $v \sim r^a$ (a negative) would allow this absorption of inner layers of the atmosphere by outer ones on the same side of the star.

¶ Stratton, *Ann. Solar Physics Obs., Cambridge*, 4, 133, 1936.

with the variation in absorption-line shape, both for *H* and *FeII* lines, with emission intensity, gives the impression that the absorption lines are not trustworthy indices of real velocities. Hence the possibility that recorded velocity differences may be spurious is not ruled out, and atoms of all elements in any one layer may, indeed, move with effectively the same velocity.* McCrea † has performed preliminary calculations which suggest that this is to be expected. He finds that for the order of density associated with a prominence the mutual drag of atoms reduces all kinds to a common velocity, though densities appreciably less than this would permit large velocities of separation.

On December 23 a new set of absorption lines of greater velocity appeared and within three days had replaced the original series. ‡ It is suggested that this resulted from a fresh outpouring of matter from a nucleus much below the photospheric layers, and that this new outrush quickly caught up remnants of the first outpouring and dragged them along. The advent of this more rapid expansion might well cause the opacity to decrease so that the effective photosphere retreated, accounting for the magnitude drop in brightness. If the transition took place in an irregular manner and left large masses of gas no longer in line of sight to the star, the observed replacement of the hydrogen absorption lines by emission peaks, equally displaced to the violet, might be accounted for.

Summary

1. The contours of absorption and emission bands of hydrogen, iron, magnesium and silicon have been measured for 18 spectrograms secured on each of the first 8 days following the outburst.

2. With the help of the Greenwich energy distribution measurements the intensities of the emission bands have been converted into true relative values. The changes with time are studied, and it is found that, except for the neutral oxygen and carbon lines, which are of abnormal strength, the variations are similar to those in the spectral sequence.

3. The Balmer decrement in emission is examined and found to be considerably slower than theory indicates for either electron capture or polychromatic radiative equilibrium. In absorption the Balmer lines decrease steadily in strength towards the series limit. The intensity of the *Hβ* absorption is so large as to suggest that self-absorption is occurring in

* On the other hand if, at the outset, relative velocities of 300 km./sec., corresponding to hundreds of volts energy, are permissible between intermingled metallic and hydrogen atoms, possibilities of high excitation phenomena through collisions arise and might go some way towards explaining the association of a B-type spectrum with a colour temperature of 10,800°.

† *M.N.*, 95, 509, 1935.

‡ Wilson (*Ap. J.*, 82, 233, 1935) suspected the presence of absorption II on the violet wing of the K line as early as December 22.6. K was then easily the strongest absorption line present, and a tracing from our plate of December 21.8 shows clearly that even at this earlier date the form of the calcium line was influenced by a second component of moderate intensity.

the emitting atmosphere. For iron the emission intensities are compared with those in the solar chromosphere. The ratio of strong to weak iron absorption lines is much larger than in ordinary stars.

4. When the widths of the red halves of the hydrogen emission bands are expressed in velocities it is found that $H\beta$ is much wider, at a given fraction of the maximum intensity, than $H\gamma$ and $H\delta$, which two lines agree excellently in shape. The effect is shown to be much too large to be of instrumental origin. This indicates that the Balmer decrement is most rapid in the most displaced part of the line, which, it is suggested, originates in the outer atmosphere of the nova where the emission is not of recombination type. For $H\gamma$ and $H\delta$ the extreme wing of the violet absorption is more displaced than that of the redward emission.

5. The mean contour of the hydrogen emission lines is very well represented by an error curve. The maxima become slightly more peaked during the period of observation. Away from the band centre good agreement is obtained with Gerasimovič's theoretical contour $I = u^{-0.43} - 1$. The mean contour for four $FeII$ lines is closely similar to that for $H\beta$, although the corresponding absorptions are less displaced than for hydrogen.

6. It is considered that the appearance of the spectra, in particular the shapes of the absorption lines, are more favourable to the hypothesis of continuous ejection than to that of a discrete expanding shell. Two models involving continuous ejection of atoms with high velocity from the photosphere are considered. Neither model will explain all the results. That suggested by the observed agreement of the contours with Gerasimovič's curve corresponds to outward deceleration of particles. As his curve neglects occultation effects, it is doubtful if it can apply so soon after the outburst. In addition, the large deceleration needs to be explained. The other model assumes constant velocity of particles, and is shown to be capable of producing the observed emission contours when occultation by the central star is taken into account.

*Solar Physics Observatory,
Cambridge.*

THE ORBIT OF OΣ 79.

L. S. T. Symms.

(Communicated by the Astronomer Royal)

OΣ 79, A.D.S. 3135, 55 Tauri, mags. 7.0–8.9, Go.

This binary has just completed one revolution in its orbit since it was first measured in 1844 by Mädler. There is a good series of observations in position-angle covering the whole period except for the seven years 1891 to 1898, when the small separation made it a difficult object. The observations in separation are unsatisfactory until 1890, when Schiaparelli and Burnham secured accordant measures before the companion closed in

to periastron. Immediately after periastron, while the separation was still less than a third of a second, we find many fine observations by Hussey, Aitken, Comstock and Van Biesbroeck which appear, if anything, more consistent than the modern observations made since the object became a comparatively easy object.

The present work is the outcome of an enquiry into various methods of double-star orbit computing by Dr. Woolley. The orbit was first computed some months ago by Russell's method,* which is more or less a graphical one, very convenient and rapid for a preliminary orbit, but when the observations of a binary warrant a more definitive orbit graphical methods are apt to leave the computer with the feeling that everything possible has not been obtained from the observations available. In the present case Dr. Woolley and the writer independently computed orbits by this method, but it was felt that the results did not justify publication, especially as Aitken's orbit of this binary† is not greatly in need of revision even after twenty-five years' additional observations. After some experience with the method of Innes and van den Bos‡ it was decided to finish the orbit by that method.

Interpolating curves were drawn of the observed position-angles, and separations and normal places were adopted independently by Dr. Woolley and the writer. After discussion of the small differences between the two sets the following places were adopted:—

Date	θ	ρ
	°	"
1859.0	35.7	...
71.9	50.3	...
82.2	62.3	...
89.9	84.2	...
1900.2	244.5	.22
05.3	281.5	.305
10.5	310.5	.335
15.6	332.1	.375
20.7	351.3	.45
25.9	3.8	.51
31.0	12.0	...

It will be noticed that no values of the separation were adopted for dates prior to 1900 on account of the unsatisfactory scatter in the observations in this co-ordinate before periastron.

The method of Innes and van den Bos consists of obtaining differential corrections to the elements of a preliminary orbit by means of a least squares solution of equations in the form §

$$\begin{aligned}\Delta x &= X\Delta A + Y\Delta F + P_x\Delta e + Q_x(n\Delta T - (t - T)\Delta n), \\ \Delta y &= X\Delta B + Y\Delta G + P_y\Delta e + Q_y(n\Delta T - (t - T)\Delta n),\end{aligned}$$

* *M.N.*, 93, 599, 1933.

† *Lick Pub.*, 12, 30, 1914.

‡ *Union Obs. Circ.*, 68, 1926.

§ Van den Bos, *loc. cit.*; also Woolley and Symms, *M.N.*, 97, 168 and 438, 1937.

Δx and Δy being residuals in x and y between the places computed from the preliminary orbit and the adopted places. Differentiating the equations $\tan \theta = y/x$ and $\rho^2 = x^2 + y^2$ gives the expressions

$$\rho \frac{\Delta \theta}{R^0} = -\sin \theta \cdot \Delta x + \cos \theta \cdot \Delta y$$

$$\Delta \rho = \cos \theta \cdot \Delta x + \sin \theta \cdot \Delta y$$

(where R^0 = degrees in a radian), which allow working in Polar co-ordinates instead of Cartesian. This is a great advantage of the method, for, as in the present case, when one co-ordinate is badly observed over part of the period, it still remains possible to form equations of condition for these dates from observation in the other co-ordinate.

The Campbell elements of the orbit by Russell's method were converted into the Innes form and, together with Aitken's orbit, were used to compute places for comparison with the adopted normal places. The former orbit gave the smaller residuals, so this was used as the basis orbit in forming the equations. The residuals were :

	$\Delta \theta$	$\Delta \rho$
	°	"
1859.0	-1.7	...
71.9	+0.3	...
82.2	-0.8	...
89.9	+0.7	...
1900.2	+0.4	-0.02
05.3	-1.7	+0.01
10.5	-2.7	+0.005
15.6	-4.0	-0.015
20.7	-1.0	-0.01
25.9	-0.4	-0.02
31.0	-1.3	...

The Innes elements of the basis orbit are :

$$\begin{array}{llll} A - .391 & F + .020 & e \ .620 & T \ 1897.6 \\ B - .222 & G - .462 & n \ 3.892 & \end{array}$$

The differential corrections to these elements obtained from the least squares solution, together with their probable errors, is as follows :—

$$\begin{array}{ll} \Delta A + .0237 \pm .0012 & \Delta e - .0063 \pm .0004 \\ \Delta B + .0183 \pm .0014 & \Delta T + .29 \pm .014 \\ \Delta F + .0059 \pm .0020 & \Delta n + .144 \pm .006 \\ \Delta G + .0006 \pm .0035 & \end{array}$$

The probable errors indicate the agreement between the corrected orbit and the normal place. They are satisfactorily small, and the correction to the eccentricity is not large enough to make a second approximation necessary.

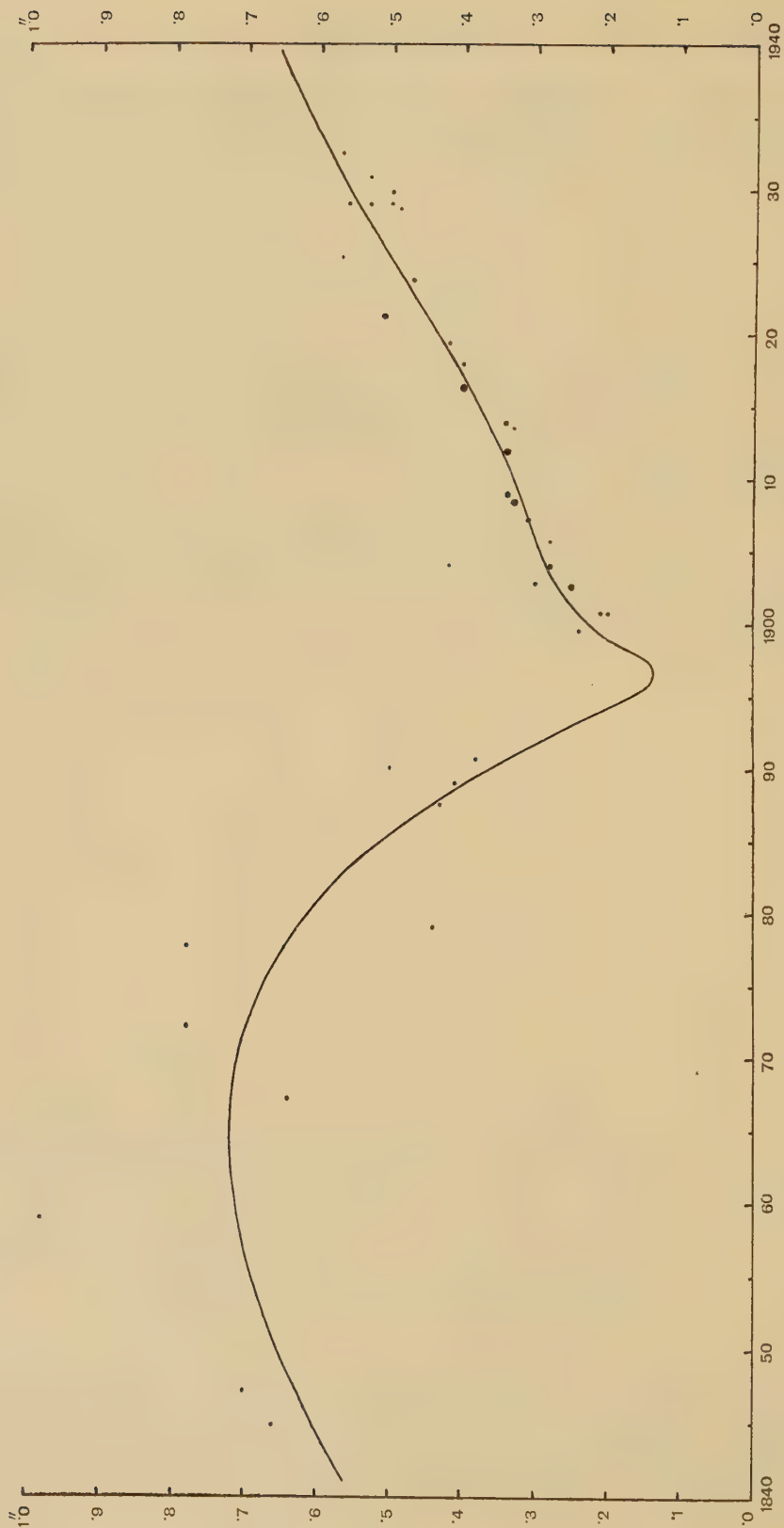


FIG. 1.—Observations of separation compared with orbit.

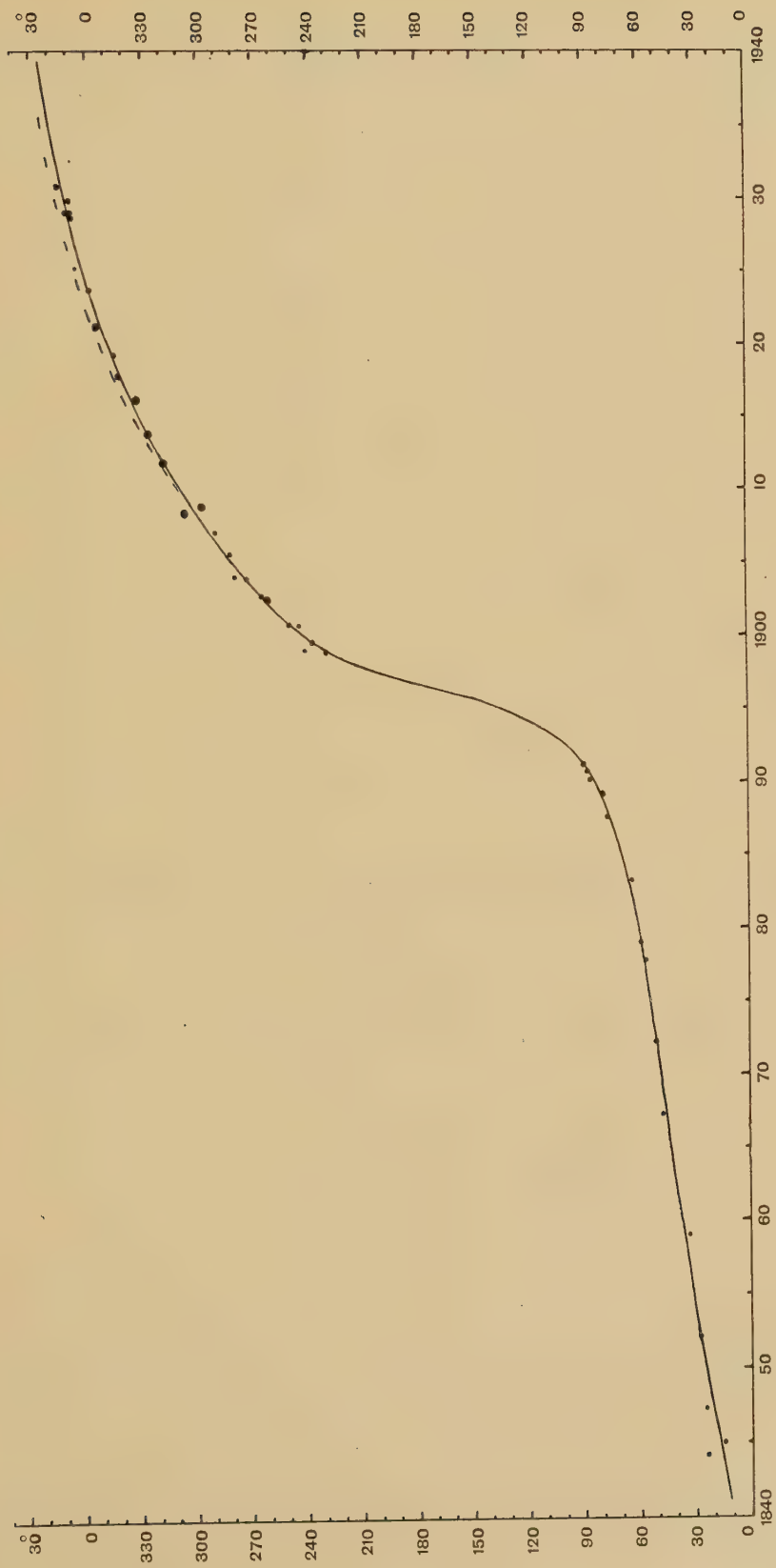
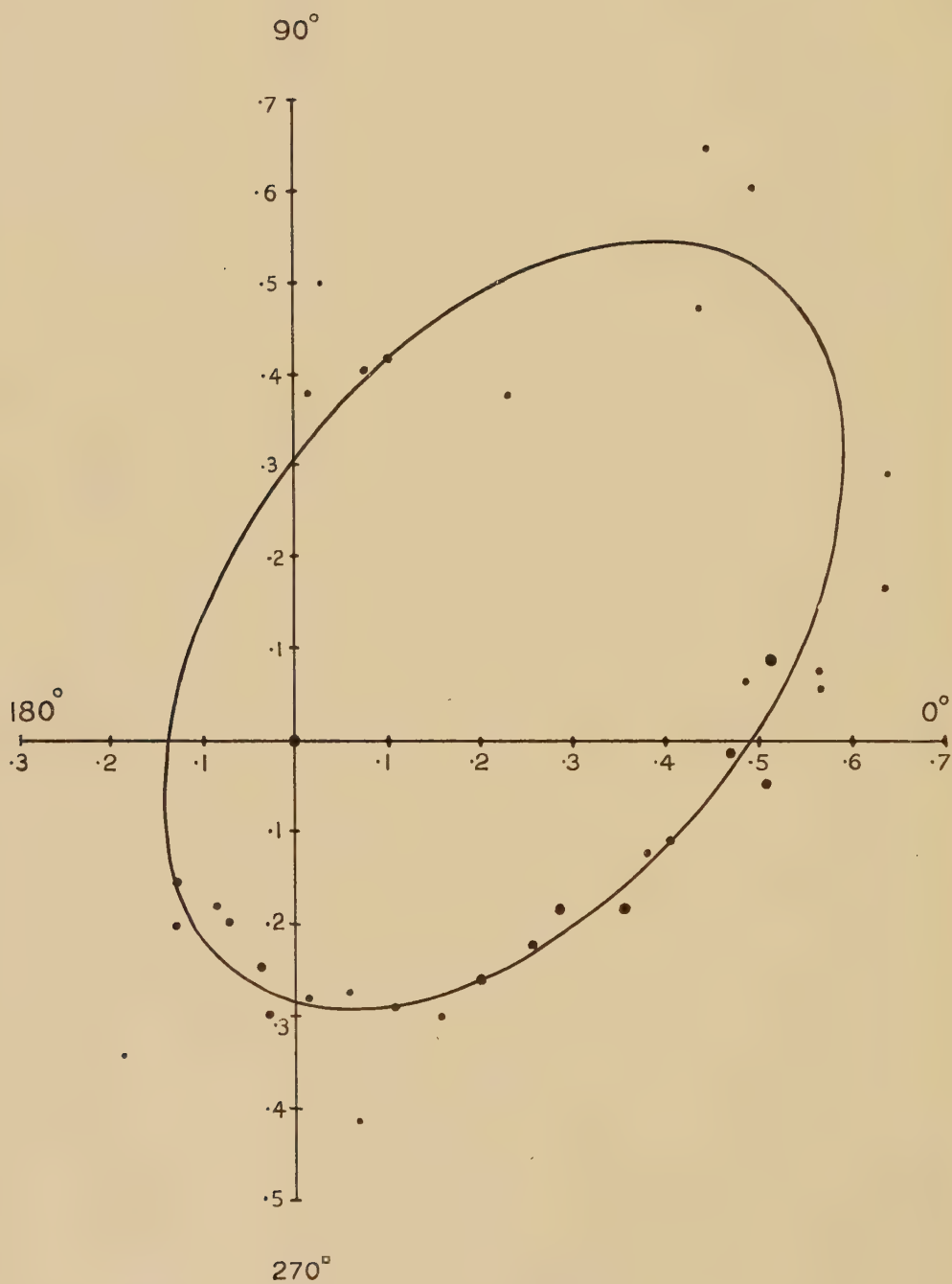


FIG. 2.—Observations of angle compared with orbit.

FIG. 3.—OΣ 79. *Apparent orbit.*

The corrected elements give the following places and residuals from the adopted normal places :—

	Computed		O - C	
	θ	ρ	$\Delta\theta$	$\Delta\rho$
	$^{\circ}$	"	$^{\circ}$	"
1859.0	35.6	...	+0.1	...
71.9	49.7	...	+0.6	...
82.2	63.4	...	-1.1	...
89.9	82.8	...	+1.4	...
1900.2	243.1	.230	+1.4	-0.10
05.3	282.1	.299	-0.6	+0.06
10.5	311.1	.336	-0.6	-0.01
15.6	333.3	.383	-1.2	-0.08
20.7	350.0	.444	+1.3	+0.06
25.9	2.9	.509	+0.9	+0.01
31.0	12.7	...	-0.7	...

The sum of the squares of the residuals $\rho \frac{\Delta\theta}{R^0}$ and $\Delta\rho$ is .00083 compared with .00319 with the basis orbit.

The application of these corrections gives the following final elements :—

$$\begin{aligned}
 A &= -\cdot 367 & e &= \cdot 614 \\
 B &= -\cdot 204 & T &= 1897.9 \\
 F &= +\cdot 026 & n &= 4.036 \\
 G &= -\cdot 461
 \end{aligned}$$

We may add for convenience

$$\begin{aligned}
 \text{Period} & 89.20 \text{ years} \\
 \text{Semiaxis major} & 0''.53 \\
 \text{Inclination} & \pm 51^{\circ}.6
 \end{aligned}$$

These elements give a dynamical parallax of $0''.021$ as compared with the spectroscopic parallax $0''.022$ (Mt. Wilson) and trigonometrical parallax $0''.046 \pm 0.006$ (Allegheny).

Ephemeris

	$^{\circ}$	"
1940.0	26.0	0.66
45.0	32.0	0.69
49.9	37.5	0.71
54.9	42.9	0.72
59.8	48.3	0.71
64.8	54.0	0.67

The figures show the observations compared with the final orbit and, in the case of the position-angles, part of Aitken's orbit shown dotted.

NOTE ON THE DYNAMICS OF A STELLAR SYSTEM.

Bertil Lindblad, Assoc.R.A.S.

In a recent highly interesting paper on the "Dynamics of a Stellar System,"* G. L. Clark states the equivalence of Eddington's and Jeans's methods of finding the frequency function of the spatio-velocity distribution in a stellar system, when this function is of the form

$$f(a^2L^2 + b^2M^2 + c^2N^2).$$

Here L , M , N are the velocity components along the principal directions of an orthogonal system of co-ordinates. Clark also contends, however, that Jeans's method, which expresses f as a function of the first integrals of the motion, in accordance with a theorem which appears to have been first given by Poincaré,† is less general than Eddington's method in the case of a distribution

$$f(a^2L^2 + b^2M^2 + c^2N^2 + 2fMN + 2gNL + 2hLM).$$

This is alleged to follow because Jeans's method does not take into account certain invariant relations which have not the character of first integrals. The conclusion was brought out by the discussion of the cases (b) and (c), representing motion in the plane in the case of a quasi-elastic field and a Newtonian potential respectively. It is my object to show, however, that the cases in question are completely covered by Jeans's method. This follows when we consider that the first integrals of the motion are really

$$I_1, I_2, \omega,$$

where I_1 is the integral of energy, I_2 the area integral in the plane of motion, and ω the longitude of the apsidal line. If we consider the subsidiary equations

$$\frac{d\varpi}{\Pi} = \frac{d\theta}{\Theta} = \frac{d\Pi}{\Theta^2 + \frac{\partial V}{\partial \varpi}} = \frac{d\Theta}{-\frac{\Pi\Theta}{\varpi}}, \quad (1)$$

we get easily the integrals I_1 and I_2 . In order to derive ω as a first integral from these equations, we form in the quasi-elastic case

$$\sin 2\theta[\Pi d\Pi - \Theta d\Theta - 2\Pi\Theta d\theta + 2E\varpi d\varpi] + \cos 2\theta[\Theta d\Pi + \Pi d\Theta + (\Pi^2 - \Theta^2 + 2E\varpi^2)d\theta],$$

which vanishes identically by (1), if $\frac{\partial V}{\partial \varpi} = -2E\varpi$. We may then write for the expression just derived

$$d[(\Pi^2 - \Theta^2) \sin 2\theta + 2\Pi\Theta \cos 2\theta + 2E\varpi^2 \sin 2\theta] = 0,$$

* *M.N.*, 97, 182, 1937.

† *Leçons sur les hypothèses cosmogoniques*, 1911, p. 100.

or

$$(\Pi^2 - \Theta^2) \sin 2\theta + 2\Pi\Theta \cos 2\theta + 2E\varpi^2 \sin 2\theta = C_1, \quad (2)$$

We find without great difficulty that

$$C_1 = \sqrt{(I_1 - 2E)^2 - 8EI_2^2} \sin 2\omega.$$

In the Newtonian case we find in a similar way

$$\sin \theta [\Theta^2 d\varpi + \varpi \Pi \Theta d\theta + 2\varpi \Theta d\Theta] + \cos \theta [-\Pi \Theta d\varpi + (\varpi \Theta^2 - \mu) d\theta - \varpi \Pi d\Theta - \varpi \Theta d\Pi] = 0,$$

if $\frac{\partial V}{\partial \varpi} = -\frac{\mu}{\varpi^2}$. We then get

$$d[(\varpi \Theta^2 - \mu) \sin \theta - \varpi \Pi \Theta \cos \theta] = 0,$$

and

$$(\varpi \Theta^2 - \mu) \sin \theta - \varpi \Pi \Theta \cos \theta = C_2, \quad (3)$$

where we may identify

$$C_2 = \mu e \sin \omega.$$

We may, of course, easily form expressions of the co-ordinates and velocities in which ω is the only arbitrary constant. In order to obtain expressions of the second order in the velocities, we observe that we have in the quasi-elastic case

$$\left. \begin{aligned} \rho \sin 2\omega &= (\Pi^2 - \Theta^2) \sin 2\theta + 2\Pi\Theta \cos 2\theta + 2E\varpi^2 \sin 2\theta \\ \rho \cos 2\omega &= (\Pi^2 - \Theta^2) \cos 2\theta - 2\Pi\Theta \sin 2\theta + 2E\varpi^2 \cos 2\theta, \end{aligned} \right\} \quad (4)$$

where

$$\rho = \sqrt{(I_1 - 2E)^2 - 8EI_2^2},$$

E being the constant in the potential function

$$\phi = -E(1 + \varpi^2).$$

In the Newtonian case

$$\left. \begin{aligned} \mu e \sin \omega &= (\varpi \Theta^2 - \mu) \sin \theta - \varpi \Pi \Theta \cos \theta \\ \mu e \cos \omega &= (\varpi \Theta^2 - \mu) \cos \theta + \varpi \Pi \Theta \sin \theta, \end{aligned} \right\} \quad (5)$$

where

$$e = \sqrt{1 + \frac{I_1 I_2^2}{\mu^2}}.$$

It follows that in both cases the most general form of the frequency function for steady motion may be written

$$f(I_1, I_2, \omega),$$

and that, in order to get the most general expression of the second order in the velocities, we have to introduce ω in f in the Newtonian case by $e \sin \omega$ and $e \cos \omega$, thus

$$f(AI_1 + DI_2 + \kappa I_2^2 + B\mu e \sin \omega + C\mu e \cos \omega), \quad (6)$$

and in the quasi-elastic case in a similar way by means of $\rho \sin 2\omega$ and

$\rho \cos 2\omega$. It is easily verified that in both cases we get results in agreement with Heckmann's * formulæ.

Considering especially the Newtonian case we may now try to find the connection with Clark's procedure. We may find a particular ω_0 such that

$$B = F \sin \omega_0, \quad C = F \cos \omega_0. \quad (7)$$

We then get

$$B\mu e \sin \omega + C\mu e \cos \omega = F[(\varpi\Theta^2 - \mu) \cos(\theta - \omega_0) + \varpi\Pi\Theta \sin(\theta - \omega_0)]. \quad (8)$$

We further have by (5)

$$\mu e = (\varpi\Theta^2 - \mu) \cos(\theta - \omega) + \varpi\Pi\Theta \sin(\theta - \omega). \quad (9)$$

Clark's procedure is to substitute in (6) according to (8). The identity with Clark's procedure follows because his "invariant" I_3 is identical with (9), and because on p. 201 of his paper he treats ω not as an arbitrary constant of integration, but as a constant parameter. His way of proceeding is thus really, and quite correctly according to (8), to substitute $F\mu e$ in place of the two last terms in (6), after setting $\omega = \omega_0$ in the expression for μe according to (9). In this way we get formulæ for the ellipsoidal constants h^2 , m , k^2 , which are actually in agreement with those resulting directly by inserting (5) in (6). Nevertheless, the introduction of the "invariant" I_3 in this way appears plainly artificial. The apsidal longitudes ω of individual orbits, of course, do not coincide with the particular value of ω which enters into Clark's equation (36), and which in the present paper is called ω_0 . In any case it is clear at once that the results obtained above remove entirely Clark's criticism against Jeans's method.

In the quasi-elastic case the relation (30) of Clark's paper (where the factor $2E$ of the second term should be transferred to the first term), which apparently does not contain any arbitrary constant, is valid only for a special choice of the co-ordinate system, namely, the one in which one of the axes coincides with the apsidal line of the orbit considered. The longitude of the apsidal line thus enters implicitly. With this choice of co-ordinate system Clark's integrals I_3 and I_4 are connected with I_1 and I_2 by the relations

$$I_3 + I_4 = I_1 - 2E, \quad I_3 I_4 = 2E I_2^2,$$

so that even when I_3 and I_4 are included the independent first integrals remain the three I_1, I_2, ω . The quantity ω in Clark's equation (26) is evidently a parameter analogous to the quantity ω_0 considered above in the Newtonian case. The formulæ (26) and (27) are equivalent with those resulting by inserting our relations (4) in the expression

$$f(AI_1 + DI_2 + \kappa I_2^2 + B\rho \sin 2\omega + C\rho \cos 2\omega).$$

It appears then, in consequence of Shiveshwarkar's † and Clark's discussions, together with the considerations just given here, that the methods

* *M.N.*, 96, 67, 1935.

† *M.N.*, 96, 749, 1936.

of Eddington and Jeans are in principle quite equivalent for distributions which are everywhere ellipsoidal.

Eddington's theorem concerning the existence of spherical systems satisfying Poisson's equation may be extended to more general frequency functions of Jeans's type. Further, the existence of oblate systems of Jeans's type which satisfy Poisson's equation, and which possess a massive nearly spherical central part, may also be shown. Such a state of things has been assumed, implicitly or explicitly, in the various stages of development of the rotation theory of the galaxy, though this theory does not have to postulate a strictly steady state. The velocity distribution may be of approximately ellipsoidal form in wide regions of the system. In the rotation theory special account is taken of the variation of the parameters of the velocity distribution with the physical characteristics of the objects. Of special interest is, for instance, the relation between internal velocity-dispersion and speed of rotation.

The Poincaré-Jeans method now allows us, as soon as an existence theorem of sufficiently wide application has been established, to construct an ideal picture of the entire stellar system as a vast flattened formation in approximate equilibrium. Eddington's method (especially the cases of cylindrical and prolate spheroidal velocity surfaces with superposed rotation) is applicable beside that of Jeans's in those regions where the distribution adheres closely to the ellipsoidal law. An actual derivation of the ellipsoidal law from more basic assumptions in the case of low internal velocity dispersion has been given by the present writer.* Several observed facts concerning the space distribution and the velocity distribution of the stars indicate, however, considerable deviations from the conditions of an ideal steady state. Of special importance are the deviations which appear to be caused by zones of instability due to the occurrence of rapid radial density-gradients and to a secularly increasing general flattening of the system. It seems that these instabilities are of the same kind as those which give rise to the spiral structure of extra-galactic nebulae. Still, for our immediate surroundings of the galactic systems it has been shown† that the essential features of the distribution, including certain plain deviations from the "normal" ellipsoid, indicate a state of approximate local equilibrium which may remain of the same type for very long time. A detailed discussion of the existence theorem for systems of Jeans's type, which, as just mentioned, has to a considerable extent been anticipated in the development of the rotation theory of the galaxy, will be given elsewhere.

*Stockholm Observatory,
Saltsjöbaden :
1937 March.*

* *Arkiv för Matematik, Astronomi och Fysik*, 20A, No. 17, 1927 (*Upsala Medd.*, No. 26) ; *Stockholms Observatoriums Annaler* 12, No. 4, 1936, p. 31.

† *M.N.*, 97, 15, 1936.

ON THE INTERNAL CONSTITUTION OF ECLIPSING BINARIES. (SECOND PAPER.)

Zdeněk Kopal, D.Sc.

In a paper recently communicated to the Society* the density condensations of eclipsing binaries deduced from their apparent ellipticities were studied in some detail. The main conclusions of this paper are in disagreement with the usually assumed stellar model; but several other reasons seem to point towards similar conclusions,† so that there can be but little doubt that they are generally correct. The question arises whether Eddington's stellar model will be ultimately found to be an inadequate representation of reality, or whether it is possible by a suitable generalisation to bring it into agreement with observational evidence. The main defects, in which the model may not correspond to reality, are

- (i) The supposed absence of the rotation,
- (ii) The supposed constancy of the molecular weight,
- (iii) The supposed constancy of $k\eta$.

The aim of the present paper will be to discuss the effects of departures from these limitations on the internal constitution of stars, and to examine whether the properties of such generalised configurations can be brought into agreement with observed facts.

Let us consider the effects of rotation first. We have the advantage of being able to use results of extended investigations on the equilibrium of distorted polytropes due to S. Chandrasekhar.‡ It has been shown by Chandrasekhar§ that if a polytrope with constant mass is set in a slow rotation its central density λ and the volume V satisfy

$$\lambda_\omega = \lambda_0 \left[1 - \frac{2n}{3-n} \left(1 - \frac{3\psi_0'(\xi_1)}{\xi_1} \right) v \right] \quad (1)$$

and

$$V_\omega = V_0 \left\{ 1 + \left[\frac{9\psi_0(\xi_1)}{\xi_1^2} - \frac{3(1-n)}{3-n} \left(1 - \frac{3\psi_0'(\xi_1)}{\xi_1} \right) \right] v \right\}, \quad (2)$$

where ξ_1 is the first zero of the Emden equation, $\psi_0(\xi_1)$ the associated Emden function of the order 0, $\psi_0'(\xi_1)$ its derivative, n the polytrope index, and

$$v = \frac{\omega^2}{2\pi G\rho}.$$

* *M.N.*, 96, 854, 1936. It will be referred to as Paper I.

† See especially S. Chandrasekhar, *M.N.*, 96, 647, 1936, where the author arrived at similar results by a method entirely different from ours.

‡ *M.N.*, 93, 390, 449, 462, 539, 1933.

§ *M.N.*, 93, 559-561, 1933.

If the mass remains constant, the mean density ρ varies as V_{ω}^{-1} and from (1) and (2) we easily get

$$\left(\frac{\lambda}{\rho}\right)_{\omega} = \left(\frac{\lambda}{\rho}\right)_0 \frac{1 - \frac{2n}{3-n} \left(1 - \frac{3\psi_0'(\xi_1)}{\xi_1}\right)v}{1 - \left[\frac{9\psi_0(\xi_1)}{\xi_1^2} - \frac{3(1-n)}{3-n} \left(1 - \frac{3\psi_0(\xi_1)}{\xi_1}\right)\right]v} \quad (3)$$

If the coefficients of v are not too large, (3) can be simplified to

$$\left(\frac{\lambda}{\rho}\right)_{\omega} = \left(\frac{\lambda}{\rho}\right)_0 \cdot \left[\frac{9\psi_0(\xi_1) + 3\xi_1\psi_0'(\xi_1)}{\xi_1^2}\right]v. \quad (4)$$

The reader will notice that the expression in brackets on the right side of (4) is always greater than unity, and therefore the density condensation must increase when the configuration is set rotating; this holds for all n except $n=3$. A glance at (3) will show the exceeding sensitivity of the coefficient when $n \sim 3$; but it is easy to see that in this case (3) reduces to

$$\left(\frac{\lambda}{\rho}\right)_{\omega} = \left(\frac{\lambda}{\rho}\right)_0, \quad (4.1)$$

i.e. for n equal or very near to 3 the density condensation is constant, independent of the (slow) rotation. For $n \neq 3$, however, the rotation involves an increase of the density condensation; and the polytrope in rotation behaves like another polytrope with a higher index. The question arises whether such changes may not be responsible for the changes of Δ discussed in Paper I.

Let us first provide the necessary numerical data. The mean densities of components of binary systems can be expressed as follows:—

$$\left. \begin{aligned} \rho_1 &= 0.01344 P^{-2} r_1^{-3} \frac{\mathfrak{M}_1}{\mathfrak{M}_1 + \mathfrak{M}_2}, \\ \rho_2 &= 0.01344 P^{-2} r_2^{-3} \frac{\mathfrak{M}_2}{\mathfrak{M}_1 + \mathfrak{M}_2}, \end{aligned} \right\} \quad (5)$$

where r_1 and r_2 are the radii of components expressed in terms of the orbital radius as unit, $\mathfrak{M}_1$ and $\mathfrak{M}_2$ are the masses of both components (the suffix 1 always referring to the more massive body), and P the orbital period expressed in solar days.

Introducing $\frac{\mathfrak{M}_2}{\mathfrak{M}_1} = q$, and transforming P into seconds, we get

$$\left. \begin{aligned} v_1 &= \frac{\omega^2}{2\pi G \rho_1} = C \cdot r_1^3 (1+q), \\ v_2 &= \frac{\omega^2}{2\pi G \rho_2} = C \cdot r_2^3 \left(1 + \frac{1}{q}\right), \end{aligned} \right\} \quad (6)$$

where

$$C = \frac{2\pi}{0.01344 \times (86.400)^2 \times 6.66 \times 10^{-8}} = 0.9403.$$

The quantities r_1 and r_2 are identical with v_1 and v_2 of the first paper ; q has already been tabulated in the same paper, so we are able to evaluate v_1 and v_2 in terms of values known from Paper I.

TABLE I

Bo-B4 stars					
	k	Δ	v_1	v_2	$\frac{v_1}{v_2}$
TT Aur	1.12	2.452	0.0582	0.0521	1.12
RZ Cen	0.50	1.363	0.0169	0.2655	0.064
U CrB	0.60	1.288	0.0059	0.0723	0.821
β Lyr	1.89	2.823	0.1596	0.0532	3.00
V Pup	1.09	2.190	0.0987	0.0992	0.996
Z Vul	0.95	2.677	0.0258	0.0656	0.394
B5-B9 stars *					
σ Aql	1.00	1.970	0.0225	0.0271	0.832
SV Cen	1.35	2.884	0.0787	0.0662	1.18
u Her	0.82	2.058	0.0316	0.0908	0.348
U Oph	1.00	2.040	0.0232	0.0331	0.700
V Ser	0.63	1.980	0.0085	0.0539	0.158
λ Tau	1.33	2.231	0.0272	0.0570	0.478
RS Vul	0.76	1.252	0.0100	0.0716	0.140
A stars					
AN And	1.00	1.850	0.0219	0.0420	0.520
S Ant	1.30	2.503	0.1162	0.0731	1.59
SX Aur	0.84	1.649	0.0534	0.0870	0.614
TV Cas	1.00	2.031	0.0278	0.0470	0.592
SZ Cen	1.25	2.295	0.0308	0.0428	0.720
TW Dra	0.65	1.264	0.0115	0.1466	0.078
RX Her	1.00	2.320	0.0149	0.0166	0.902
V Tri	1.25	2.033	0.0662	0.0487	1.33
F stars					
RS CVn	0.80	1.116	0.0546	0.1154	0.473
RV CVn	1.00	1.708	0.0577	0.0577	1.000
U Peg	1.00	1.680	0.1107	0.1361	0.710
AB Per	0.55	1.199	0.0135	0.0778	0.174
TY Pup	1.00	1.472	0.0308	0.0453	0.680
W UMa	1.50	1.936	0.1698	0.0701	2.42
G-K stars					
SS Cam	2.97	2.537	0.0969	0.0092	10.50
RZ Cnc	0.48	0.939	0.0155	0.2780	0.557
RW Com	0.90	1.347	0.0807	0.1160	0.695
W Cru	2.74	2.471	0.1404	0.0157	8.95
RT Lac	1.00	1.306	0.0412	0.0503	0.820
SW Lac	0.93	1.541	0.0729	0.1592	0.458
U Sct	1.47	1.806	0.0828	0.0510	1.62

* Taking this opportunity, the writer would like to correct two printing errors which occurred in the Table in Paper I : in the B5-B9 star group, for δ Aql and μ Her read σ Aql and u Her.

Table I contains the results.* The individual columns show (1) the name, (2) the radii ratio $k = \frac{r_1}{r_2}$, (3) the values Δ †—both taken from Paper I, (4) and (5) the values v_1 and v_2 , and (6) their ratio $\frac{v_1}{v_2}$.

The comparison between the density concentrations as specified by Δ and v is summarised in Table II in detail. The stars have been divided

TABLE II

Δ	$\bar{\Delta}$	v_1	v_2	$\bar{v}$	n
0.900–1.400	1.230 $\pm$ 0.096	0.0278 $\pm$ 0.0273	0.1326 $\pm$ 0.0649	0.0802 $\pm$ 0.0461	9
1.401–1.800	1.610 $\pm$ 0.083	0.0651 $\pm$ 0.0214	0.1009 $\pm$ 0.0452	0.0830 $\pm$ 0.0666	5
1.801–2.200	1.989 $\pm$ 0.081	0.0553 $\pm$ 0.0393	0.0563 $\pm$ 0.0183	0.0558 $\pm$ 0.0288	10
2.201–2.600	2.401 $\pm$ 0.103	0.0692 $\pm$ 0.0417	0.0381 $\pm$ 0.0208	0.0536 $\pm$ 0.0312	7
> 2.600	2.795 $\pm$ 0.079	0.0880 $\pm$ 0.0477	0.0617 $\pm$ 0.0056	0.0748 $\pm$ 0.0266	3

in five groups according to Δ ; the individual columns give (1) the range of Δ , (2) its mean value in this range, (3) the mean v_1 , (4) the mean v_2 , (5) the mean v of the whole system, and (6) the number of stars used. The values Δ refer to the system as a whole, *i.e.* are the mean values of Δ 's corresponding to each component, and we have accordingly to compare them with the mean values of v as given in the column (5). The result of such a comparison is rather disconcerting. In disagreement with the theoretical expectations, there seems to be no evident relation between $\bar{\Delta}$ and $\bar{v}$ in any sense, and so we have to admit that the changes in the density distribution following from the theory of Chandrasekhar are minor, or are counterbalanced by other factors which control the internal constitution of stars and among which the rotation plays only an unimportant part.

Before pursuing this question further, let us examine the relation between k and v , which will yield us some interesting results. The necessary data are contained in Table III. The material was again divided in 7 groups containing stars the radii-ratios of which lie within the limits indicated in

TABLE III

k	$\bar{k}$	v_1	v_2	n
0.40–0.60	0.53 $\pm$ 0.04	0.0129 $\pm$ 0.0035	0.1734 $\pm$ 0.0988	4
0.61–0.80	0.71 $\pm$ 0.07	0.0212 $\pm$ 0.0170	0.0966 $\pm$ 0.0344	4
0.81–1.00	0.96 $\pm$ 0.05	0.0439 $\pm$ 0.0219	0.0710 $\pm$ 0.0404	14
1.01–1.20	1.10 $\pm$ 0.01	0.0784 $\pm$ 0.0202	0.0756 $\pm$ 0.0235	2
1.21–1.40	1.30 $\pm$ 0.05	0.0638 $\pm$ 0.0278	0.0576 $\pm$ 0.0097	5
1.41–1.60	1.48 $\pm$ 0.02	0.1263 $\pm$ 0.0435	0.0605 $\pm$ 0.0095	2
> 1.60	2.53 $\pm$ 0.43	0.1323 $\pm$ 0.0236	0.0260 $\pm$ 0.0181	3

* All stars from Table I are used, with exception of two systems AB And and RW Dor (both belonging to the G–K star group), where the mass-ratios, arrived at by an indirect method, seem to be too small and rather dubious.

† Δ , which in the following will be taken as an indication of the internal density condensation, is (very nearly) the constant of proportionality in the relation $\frac{2}{3}e \sim qv^3 - e$, being the oblateness of the star in question. For the exact definition of Δ , the reader is referred to Paper I, p. 856, eq. (5).

column (1). Column (2) gives the mean value of k in this range, (3) and (4) the corresponding mean v_1 and v_2 , and (5) the number of stars used.

At a glance we recognise the quite different behaviour of the two components. In a previous paper* the writer has shown that the decrease of k can properly be taken as an indication of the course of evolution of the system, because it is closely connected with the increase of orbital dimensions.† We see now, so far as we can follow it, that the brighter component starts with a fairly large v_1 (possibly not very far from the critical value at which fission took place) and this diminishes in the course of evolution. The secondary has started with a comparatively small value of v_2 , the effect of rapid rotation being counterbalanced by its high density. As the separation of the components increases, the orbital period increases too; but the mean density of the companion must simultaneously diminish so rapidly that v_2 increases—somewhere at $k=1.1$ it is already equal to v_1 , and later exceeds it many times. For small values of k the situation has completely changed: while the brighter component has attained a considerable degree of stability and may resemble a simple star, v_2 steadily increases and finally reaches such large values that the stability of the secondary may be seriously affected. The changes of $\frac{v_1}{v_2}$ with k are diagrammatically shown in fig. 1.

But what can cause this striking diminution of density of the secondary? The only admissible explanation is that it expands. In the paper already referred to,‡ in discussing the material consisting of a large number of systems, the writer has shown that other similar reasons lead to exactly the same conclusion. Now we are able to explain it very naturally as a direct consequence of the volume variations of rotating polytropes when v changes. Let us turn back to (2). It turns out that when a polytropic configuration with constant mass is set rotating, its volume must necessarily expand if v increases and *vice versa* in the whole range of n (with the exception of a narrow interval $3 < n < 3.325$, where actually a contraction takes place).§ In the present discussion we have just found that for components of eclipsing binary systems v_1 varies inversely to v_2 in the course of evolution of the system. Provided that stars are rotating polytropes, an adjustment of their volumes would be expected, in such a way that the contraction of the brighter component is accompanied by the expansion of its companion—which we actually find to be the case. This explains, by the way, why k diminishes in the course of evolution. Thus the observational evidence shows || that real stars behave in this respect as if they were polytropic configurations.

* *Zeitschr. für Astrophys.*, 9, 239, 1935.

† *Ibid.*, p. 249, fig 5.

‡ *Ibid.*, 9, 239, 1935.

§ *M.N.*, 93, 561, 1933.

|| The writer would like to point out another piece of evidence directed towards the same conclusion. Any contraction or expansion of a real star must be accompanied by a proper adjustment of its temperature; the contraction of the bright component should raise its surface temperature, while the expansion of its companion should lead to the contrary. But this also is exactly in agreement with observations. The reader will remember that in a majority of eclipsing binaries the more massive

Now let us turn back to the stellar density condensations and, having found the rotation to be inadequate to account for observed facts, consider the two other possibilities mentioned at the outset of this paper.

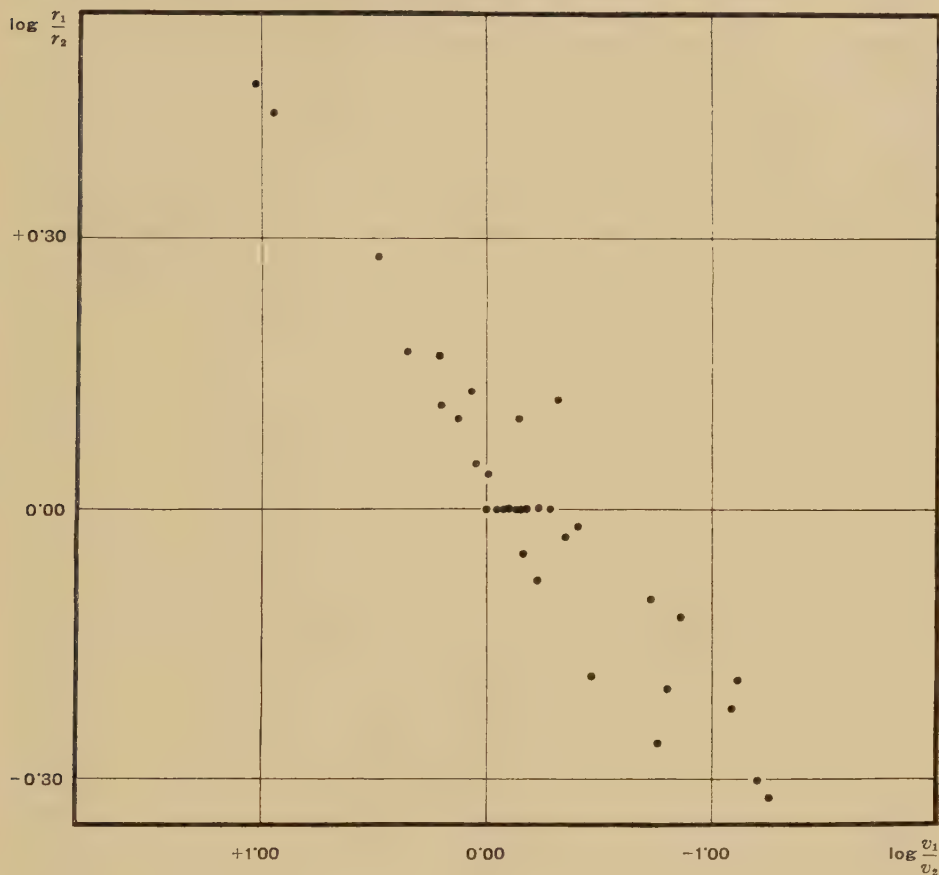


FIG. 1.

The effect of variable molecular weight in stellar interiors has already been discussed by A. S. Eddington,* who found that, if μ varies as T^{-s} , the component seems to retain a comparatively high temperature (spectral class A) through the whole course of its evolution, whilst the temperature of the secondary seems to fall off in a striking way, so that in later stages of evolution we usually are concerned with components of very unequal temperatures; the ratio of temperatures, apparently unity at the moment of fission, tends to approach small values with decreasing k , as has been shown by the writer (*Zeitschr. für Astrophys.*, 9, 248, fig. 3). This causes a rather peculiar position of eclipsing binaries in the Hertzsprung-Russell diagram (*ibid.*, p. 255, fig. 8). Most bright components are situated in the upper part of the main sequence, whilst the majority of secondaries are at its lower part, extending the limits of the main sequence far in the direction of low temperatures. All this finds its natural explanation if we accept the volume changes; the low temperatures of secondaries are consequences of their expansion, and the behaviour of bright components may be due to their contraction, which tends to maintain their temperatures. The writer would like to suggest that the occurrence of "subgiants," known only from binary pairs, might be explained in this way.

* I.C.S., § 94.

configuration conserves its polytropic structure, but its index is

$$n = \frac{3-s}{1+s}.$$

In Paper I we found that, in the majority of systems discussed, Δ 's indicate density condensations corresponding to values $1 > n > 0$. If the variable molecular weight could alone be responsible for this, s should be greater than 2, which seems unlikely. According to the investigations of Fowler and Guggenheim,* in the conditions prevailing in stellar interiors the value s might be expected, which is far from accounting for observed facts.

Let us consider the second alternative, namely, that $k\eta$ is not constant throughout the stellar interior,† and put it equal to such a function of r that we may write

$$k\eta \sim P^\epsilon,$$

where ϵ is an arbitrary quantity. Let us put further

$$\left. \begin{aligned} P_R &= \lambda P^{1+\epsilon}, \\ P_G &= P[1 - \lambda P^\epsilon], \end{aligned} \right\} \quad (7)$$

where, as usual,

$$P = P_R + P_G = \frac{1}{3}aT^4 + \frac{R}{\mu}\rho T \quad (7.1)$$

and λ is a quantity defined by (7); in the case $\epsilon = 0$ it reduces to the value $(1 - \beta)$ well known from Eddington's model, but in general, for $\epsilon \neq 0$, it is not a pure number and we do not assume that it will be constant through the interior.

Inserting (7.1) in (7) and eliminating T we get

$$P^{3-\epsilon}[1 - \lambda P^\epsilon]^4 = \frac{3\lambda R^4 \rho^4}{a\mu^4}. \quad (8)$$

Hence

$$\rho = \frac{\mu}{R} \left(\frac{a}{3\lambda} \right)^{\frac{1}{4}} \left(P^{\frac{3-\epsilon}{4}} - \lambda P^{\frac{3(1+\epsilon)}{4}} \right), \quad (9)$$

a relation specifying configurations of the type

$$\rho = K_1 P^{\frac{n_1}{n_1+1}} - K_2 P^{\frac{n_2}{n_2+1}},$$

where

$$n_1 = \frac{3-\epsilon}{1+\epsilon},$$

$$n_2 = \frac{3(1+\epsilon)}{1-3\epsilon}$$

and

$$K_1 = \frac{\mu}{R} \left(\frac{a}{3\lambda} \right)^{\frac{1}{4}} = \frac{1}{\lambda} K_2.$$

Putting $\epsilon = 0$ we obtain a complete polytrope specified by $n = 3$ (Eddington's

* *M.N.*, 85, 939, 1925.

† The writer is much indebted to the referee of this paper for valuable suggestions concerning this paragraph.

model), but for any other value of ϵ , $n_1 \neq n_2$, and, moreover, the quantities K_1 and K_2 are not constant, varying as $\lambda^{-\frac{1}{2}}$ and $\lambda^{\frac{1}{2}}$. The resulting configurations are thus of a rather complicated structure which we might properly call composite quasi polytropes.

The question arises now about the density condensations of such configurations. It is difficult to give any accurate answer without entering into detailed calculations, but it seems that the central condensation of such configurations will rapidly increase. For instance, if $\epsilon = \frac{1}{10}$, $n_1 = 2.64$, but $n_2 = 4.71$, and the resulting density condensation of this configuration will probably be much higher than that of a polytrope $n=3$. In brief, it follows that if $k\eta$ is not constant through the interior of a star but decreases outwards, the initially polytropic configuration gets distorted in structure but its density condensation tends rapidly to increase—and so the variations of $k\eta$ are inadequate to account for the facts observed at eclipsing binaries.

Mention may be made that even if $k\eta$ varies as P^ϵ , it is possible to reconcile it to some extent with Eddington's model. Assuming a definite expression for the absorption coefficient k , ϵ has bearing on the distribution of energy sources in the interior of a star and may be taken into account by properly adjusting the constant α (in Eddington's notation), which is germane to the concentration of energy sources in Eddington's model.

It is interesting to note in this connection that the rotating stars cannot generally be regarded as complete polytropes, even if they rotate with constant ω , *i.e.* as rigid bodies. The reasons are obvious.

The fundamental equations of equilibrium of rotating stars are

$$\left. \begin{aligned} \frac{dP}{dr} &= \rho \left(\frac{\partial V}{\partial r} + \frac{\partial \Omega}{\partial r} \right), \\ \frac{dp_R}{dr} &= -\frac{k\rho H}{c}, \end{aligned} \right\} \quad (10)$$

where V and Ω denote the gravitational and centrifugal potential respectively and $H = \frac{L_r}{4\pi r^2}$. After some elementary transformations eq. (10) yields

$$\frac{dp_R}{dP} = \frac{1}{4\pi c G} \cdot \frac{k}{\tau} \cdot \frac{L_r}{\mathfrak{M}_r}, \quad (11)$$

where

$$\tau = 1 + \frac{\partial \Omega}{\partial r} : \frac{\partial V}{\partial r}. \quad (12)$$

If v and therefore the oblateness of the configuration are very small, we may put

$$\frac{\partial \Omega}{\partial r} : \frac{\partial V}{\partial r} = -v, \quad (13)$$

and inserting it in (11) we get

$$\frac{L_r}{\mathfrak{M}_r} \sim \left(1 - \frac{\omega^2}{2\pi G \rho} \right), \quad (14)$$

a result generally known as von Zeipel's theorem.

So even if we assume $k\eta = \text{const}$, $\frac{dp_R}{dP}$ is not constant through the interior, but varies as τ^{-1} . Integrating (11) and using (7.1) we again get a relation of the type $P = K \cdot \rho^\kappa$, but K ceases to be constant, being a function of v and therefore of ω and ρ . The presence of the density will affect the exponent κ , so that the rotation involves a complete departure from the structure which originally might have been polytropic, and K and κ , or rather their mean values throughout the interior, may attain different values depending on the properties of each individual star.

Equation (11) can, of course, satisfy the real conditions only in case of a very slow rotation. As soon as v gets larger, the spheroidal or later ellipsoidal form of the configuration must be taken into account, and the problem becomes more intricate but unchanged in its main feature, and the energy generation per mass unit remains in some way connected with the angular velocity even for more rapid rotation as well.

When v is getting large, τ must steadily diminish and at a certain v it must reach zero. When this occurs we are allowed to put

$$\frac{\partial \Omega}{\partial r} + \frac{\partial V}{\partial r} = 0. \quad (15)$$

The hydrodynamical equation reduces in this case to

$$\frac{dP}{dr} = 0, \quad (16)$$

which has the integral

$$\frac{R}{\mu} \rho T + \frac{1}{3} a T^4 = \text{constant}. \quad (17)$$

The critical moment at which (15) is fulfilled is the moment of fission, and (17) must therefore refer to a new-born secondary. It can be satisfied only if $\frac{\partial T}{\partial r}$ and $\frac{\partial \rho}{\partial r}$ both are zero, even if the temperature is so low that the radiation pressure can be neglected. It follows that the density and temperature must for a short time remain constant or nearly constant through the interior of the new-born secondary and probably also in the far interior of the primary. This seems to be the theoretical foundation for the empirical statement put forward in the summary of Paper I: * "The components of a new-born binary are homogeneous or nearly homogeneous."

Conclusion.—The aim of the present paper was to discuss the results empirically deduced in Paper I and to consider whether these results could be related to the rotation of the stars or perhaps to some other effect. We have found that certain effects (the volume changes, temperature conditions) can properly be explained as properties of rotating polytropes—though whether the stars themselves are rotating polytropes is possibly open to question.

* *M.N.*, 96, 862, 1936.

As to the density condensations, we felt in a rather unpleasant situation. In Paper I we found that

- (i) the central condensations of stars generally are but weak,
- (ii) the condensations vary from star to star according to its evolution stage,

and the directness of interpretation of observed facts leaves no room for doubt that they are generally correct. But this is nearly the contrary of all theoretical expectations. The theory demands that the density condensation should increase with increasing v^* —and the values Δ and v , both deduced directly from observations, have been found to be explicitly independent. On the other hand, observations suggest that stellar density condensations generally are but weak, while the stellar model usually assumed up to the present leads to a high degree of density condensation, and no reasonable generalisation can establish an agreement with the observational evidence without conflicting with the physical evidence or with certain familiar conceptions which we are scarcely willing to abandon. Provided that there is no misinterpretation of observed facts, it is hard to resist an impression that there is something wrong with the theory and that the usually assumed stellar model is inadequate to the reality. Anyhow, on a closer examination, the whole complex of phenomena appears to be of a highly intricate nature, and the latent fascinating possibilities of further development in dealing with these questions are still far from being exhausted.

1423, u Pernikářky,
Praha XVI :
1937 April.

OBSERVATIONS OF NOVÆ, 1936-1937.

W. H. Steavenson.

The observations which follow were, unless otherwise stated, made with the 20½-inch Hindle reflector. References to the comparison stars used have in most cases been published in earlier papers, and especially in *M.N.*, 92, 720, 1932.

Nova Aquilæ (1918)

Observed on one night, 1936 July 17, when the magnitude was estimated to be 10.8. There has been little change in the light of this nova for several years.

* It is to be stressed that the results following from the theory of rotational distortions are directly applicable to the reality, because the tidal forces (as far as we can follow it) do not produce any re-arrangement in the internal density distribution.

Nova Aquilæ (1925)

Observed on one night, 1936 July 26, the magnitude being estimated at 15.0. There has been little, if any, change since 1934. A chart of the field, with magnitudes of comparison stars, will be found in *M.N.*, 95, 640, 1935.

Nova Aquilæ (1936 No. 1)

Observed on thirty-seven nights. Magnitudes of the comparison stars were taken from the Draper Catalogue. The observations indicate that the nova was at its brightest at the beginning of October, but this merely marks the peak of what was a very protracted maximum, lasting for several months. During this period there were rapid and considerable oscillations, both before and after the date of discovery. In this respect the nova recalls the behaviour of *Nova Pictoris* (1925), which rose quite slowly, with fluctuations, to its principal maximum.

The 3-inch finder was generally used when the nova was brighter than 8^m.0.

Date	Mag.	Date	Mag.	Date	Mag.
1936 Sept. 21	8.4	1936 Oct. 11	8.6	1936 Nov. 9	8.6
22	8.4	15	8.1	10	8.4
23	8.8	16	8.2	14	8.3
28	7.5	18	8.0	21	7.9
29	7.8	19	8.4	30	8.3
Oct. 3	7.1	21	8.7	Dec. 1	8.5
4	7.1	23	8.7	4	8.3
5	7.2	24	8.1	5	8.4
6	7.5	25	8.0	6	8.7
7	8.3	27	7.6	7	8.9
8	8.8	Nov. 5	7.9	15	9.0
9	8.7	7	8.9		
10	8.5	8	9.0		

Nova Aquilæ (1936 No. 2)

Observed on twenty-one nights. Unlike Tamm's earlier nova, dealt with above, this object appears to have had a short and sharply defined maximum, from which it was already falling at the time of its discovery. The fall was continued pretty steadily during the period covered by the observations. Magnitudes of the comparison stars were taken from the Draper Catalogue.

Date	Mag.	Date	Mag.	Date	Mag.
1936 Oct. 21	8.0	1936 Nov. 8	9.1	1936 Dec. 5	10.3
23	8.2	9	9.0	6	10.4
24	8.2	10	9.2	7	10.4
25	8.1	14	9.3	13	10.5
27	8.3	21	9.3	15	10.8
Nov. 5	8.9	Dec. 1	10.1	19	10.3
7	9.1	4	10.3	21	10.9

Nova Aurigæ (1891)

Observed on four nights. This nova is nearly steady in its light, though the observations indicate, as in former years, slight fluctuations with a range of about 0^m.2

Date	Mag.
1936 Oct. 14	14.9
19	14.9
Nov. 10	14.8
1937 Feb. 9	15.0

Nova (T) Coronæ (1866)

Observed on one night, 1936 May 1, when the magnitude was estimated at 10.01. There has been no sensible change in the brightness of this nova for many years.

Nova Cygni (1876)

Observed on fourteen nights. The slight but definite variability noted in previous years is again shown by the observations, the extreme range being 0^m.6.

Date	Mag.	Date	Mag.	Date	Mag.
1936 July 17	14.50	1936 Aug. 20	14.91	1936 Sept. 11	15.00
24	14.66	22	15.00	13	15.00
26	14.65	24	15.00	14	15.10
Aug. 15	14.91	28	15.10	19	14.91
16	15.00	Sept. 9	15.00		

Nova Cygni (1920)

Observed on four nights. The light of this nova appears to be steady, and there has been little, if any, fading during the past two or three years.

Date	Mag.
1936 Apr. 17	15.5
18	15.5
May 19	15.5
28	15.5

Nova Geminorum (1912)

Observed on one night, 1936 October 14, when the magnitude was estimated to be 14.7. The general fading of this nova has long ago ceased, and there has been no appreciable change of brightness for a number of years.

Nova Herculis (1934)

Observed on seventy-seven nights. The observations show a slow and fairly steady decline from the protracted secondary maximum reached in the summer and autumn of 1935. The fall in twelve months has been just about one magnitude. The vivid blue-green colour, observed in 1935, has persisted throughout the period covered by the observations. The latter were all made with the 3-inch finder, or else with the 43-mm. inverting monocular. The magnitudes adopted for the comparison stars were those given by Mitchell and Wirtanen in *A.J.*, 1938.

Date	Mag.	Date	Mag.	Date	Mag.
1936 Apr. 17	7.2	1936 July 25	7.7	1936 Oct. 4	7.6
18	7.1	26	7.6	5	7.6
20	7.2	29	7.7	6	7.6
22	7.1	Aug. 1	7.6	11	7.6
26	7.2	4	7.3	15	7.6
May 1	7.2	8	7.5	16	7.7
2	7.3	13	7.4	18	7.8
11	7.2	14	7.6	19	7.7
13	7.2	16	7.6	25	7.7
14	7.2	22	7.6	Nov. 5	7.7
15	7.3	23	7.5	9	7.7
17	7.2	26	7.6	10	7.8
18	7.2	27	7.6	14	7.8
19	7.3	28	7.6	21	7.7
21	7.2	Sept. 2	7.5	30	7.7
28	7.3	3	7.7	Dec. 4	7.7
June 5	7.3	7	7.6	6	7.9
9	7.2	8	7.5	13	7.9
16	7.4	9	7.6	15	7.8
17	7.4	10	7.6	19	7.8
July 1	7.5	11	7.6	21	7.8
2	7.5	13	7.7	27	7.8
5	7.5	14	7.6	1937 Feb. 19	8.1
6	7.4	15	7.6	Mar. 11	8.0
9	7.4	19	7.6	Apr. 12	8.0
24	7.6	28	7.5		

Nova Lacertæ (1910)

Observed on three nights. There has been no appreciable change in the light of this nova for many years.

Date	Mag.
1936 Apr. 17	14.3
July 17	14.3
Aug. 16	14.4

Nova Lacertæ (1936)

Observed on eighty-six nights. The observations, which were made with the 43-mm. monocular, 3-inch finder or 20 $\frac{1}{2}$ -inch reflector, according

to the brightness of the nova, indicate a fall, with slight fluctuations, from a sharp-pointed maximum on the morning of 1936 June 20.

Date (U.T.)	Mag.	Date (U.T.)	Mag.	Date (U.T.)	Mag.
1936 June 20.0	2.16	1936 Aug. 14.0	7.75	1936 Oct. 15.8	8.56
20.9	2.46	14.9	7.78	18.7	8.61
22.0	2.6 ±	15.9	7.60	19.8	8.71
22.9	3.0 ±	16.9	7.70	24.8	8.76
24.0	3.47	20.9	7.80	Nov. 6.0	8.71
28.0	4.60	22.9	7.81	7.0	8.71
30.0	4.86	23.9	7.75	10.7	8.86
July 2.9	5.41	24.9	7.65	14.0	8.91
4.9	5.61	25.9	7.75	15.0	9.06
6.0	5.71	27.9	7.89	21.7	9.31
7.0	5.81	28.9	7.84	30.9	9.31
10.0	6.09	30.0	7.84	Dec. 6.7	9.3
13.9	6.33	Sept. 2.9	8.10	7.7	9.3
15.9	6.22	3.8	7.95	16.9	9.5
17.9	6.30	4.9	7.89	22.0	9.6
21.0	6.42	8.0	8.01	25.0	9.6
22.0	6.42	8.9	8.06	31.0	9.6
22.9	6.37	9.8	8.16	1937 Jan. 1.9	9.7
24.0	6.42	10.8	8.16	7.9	9.6
24.9	6.42	11.8	8.02	10.9	9.6
26.0	6.47	13.9	8.16	12.0	9.5
26.9	6.52	19.8	8.31	17.0	9.5
30.0	6.64	28.8	8.36	18.9	9.6
Aug. 2.0	7.02	29.8	8.41	19.9	9.6
4.0	7.16	Oct. 3.8	8.36	21.8	10.0
4.9	7.16	4.8	8.41	Feb. 1.8	10.1
5.9	7.21	5.8	8.41	6.8	10.1
8.9	7.46	6.8	8.46	9.8	10.2
9.9	7.41	11.8	8.51		

Magnitudes adopted for the comparison stars up to the end of 1936 November were taken from the Draper Catalogue. After this a sequence issued by the Variable Star Section of the British Astronomical Association was used.

Nova Lyræ (1919)

Observed on four nights, 1936 April 17, 18, May 19, 28, on all of which the magnitude was estimated at 15.2. The nova is thus slightly fainter than in 1935, but no short-period variability is indicated. A chart of the field, with a list of comparison stars, was published in *M.N.*, 95, 642, 1935.

Nova Ophiuchi (1848)

Observed on twenty-two nights. The observations show a continuance of the slight variability noted in previous years, the range amounting to 0^m.6. The material is insufficient to establish any definite period for the oscillations.

Date	Mag.	Date	Mag.	Date	Mag.
1936 Apr. 17	12.4	1936 June 9	12.7	1936 July 26	12.7
18	12.2	15	12.6	Aug. 16	12.2
May 13	12.6	16	12.6	20	12.1
15	12.6	17	12.5	24	12.3
18	12.5	23	12.4	28	12.5
19	12.6	July 5	12.1	Sept. 3	12.5
28	12.5	17	12.6		
June 2	12.5	24	12.7		

Nova Persei (1901)

Observed on seventy-seven nights. The behaviour of this nova in the period 1936-1937 has been quite abnormal. During the past twenty years the variability has usually been most marked, the range being between 1^m.0 and 2^m.0. But the observations given below indicate an extreme range of only 0^m.72, while the great majority of estimates lie between the narrow limits 12.54-12.74. It does not seem possible to assign any definite period to the variations.

Date	Mag.	Date	Mag.	Date	Mag.
1936 Sept. 13	12.53	1936 Nov. 2	12.74	1937 Jan. 7	12.69
14	12.82	5	12.43	10	12.74
15	12.82	6	12.54	11	12.74
19	12.74	9	12.74	16	12.69
21	12.84	10	12.59	18	12.74
28	12.74	13	12.64	19	12.74
29	12.64	14	12.59	21	12.64
Oct. 3	12.59	20	12.59	23	12.69
4	12.64	21	12.64	30	12.79
5	12.69	30	12.64	Feb. 1	12.69
6	12.64	Dec. 1	12.69	6	12.74
7	12.64	2	12.59	9	12.64
12	12.64	4	12.69	10	12.59
13	12.64	5	12.74	11	12.53
14	12.69	6	12.64	16	12.17
15	12.74	7	12.74	17	12.69
18	12.64	13	12.84	19	12.69
19	12.74	15	12.84	22	12.74
21	12.64	16	12.69	26	12.74
24	12.84	19	12.79	27	12.79
25	12.74	21	12.69	Mar. 18	12.69
26	12.64	23	12.54	19	12.84
27	12.64	27	12.64	28	12.74
28	12.79	29	12.79	Apr. 11	12.84
30	12.64	30	12.54	12	12.89
31	12.74	1937 Jan. 1	12.69		

Nova Vulpeculæ (1670)

The faint variable star found near the reputed place of this nova in 1934 (*M.N.*, 95, 78, 1934) was observed on one night, 1936 July 17. The magnitude was estimated at 15.5, so that there has been no substantial change in brightness during the past two years.

PARTIAL TRANSIT OF MERCURY, 1937 MAY 11.
OBSERVATIONS MADE AT THE UNION OBSERVATORY,
JOHANNESBURG.

(Communicated by the Union Astronomer)

The partial transit of Mercury was observed under good conditions. The observations made were as follows :—

Telescope	Observer	Local Sidereal Time	
		External Contact Ingress	External Contact Egress
26½-inch	van den Bos, Finsen	h m s 1 48 30	h m s 2 28 50
9-inch	Johnson	1 48 48	2 29 5
Franklin - Adams Star Camera 6" guider	Wood	1 48 36	2 28 55
Franklin - Adams Twin Telescope, 6-inch	Tannenbaum	...	2 28 45

The unweighted means of these times give the results—

External contact at ingress . . . 8^h 41^m 28^s U.T.

External contact at egress . . . 9^h 21^m 37^s U.T.

The following notes were made :—

26½-inch Telescope.—Objective stopped down to 12 inches ; power 410 ; observation made by projection on screen ; limb boiling and very blurred ; position angle at maximum phase measured as 154° ; rough micrometer measures of segment of Mercury on disc gave 5".4.

9-inch Telescope.—Observation made by projection ; Mercury well defined on Sun's limb when first seen.

Franklin-Adams Star Camera.—Observation direct using solar eyepiece ; definition fair and limb steady at ingress ; at mid-phase the appearance was that of a clean-cut, semicircular bay with sharp edges ; no halo was seen ; limb boiling slightly at egress.

*Union Observatory,
Johannesburg :
1937 May 11.*

OCCULTATIONS OF STARS BY THE MOON, OBSERVED AT THE OBSERVATORY OF THE GORKI ASTRONOMO- GEODESICAL SOCIETY IN THE YEAR 1936.

K. K. Dubrovsky.

(Communicated by the Secretaries)

Date	U.T.	Star	Mag.	Pheno- menon	Moon's Limb	Observer
1936	h m s					
Feb. 27	19 15 57.9	47 Arietis	5.8	D	Dark	Professor K. Dubrovsky
Mar. 26	16 56 54.8	Berl. B. 1039	8.6	"	"	"
28	17 31 3.1	121 Tauri	5.3	"	"	"
30	16 29 31.4	61 Geminorum	5.9	"	"	"

Instrument : Zeiss refractor, 130-mm. aperture, magnifying power 80.

Observations made with eye and ear. The time corrections of the chronometer were found from the Bordeaux, Rugby and Nauen wireless signals.

Co-ordinates of Observatory { Longitude $2^{\text{h}} 56^{\text{m}} 2^{\text{s}}$ East of Greenwich.
(approximate) { Latitude $56^{\circ} 19'.6$ N.
Altitude 165 m.

Gorki, U.S.S.R., 1937.

ERRATUM

M.N., 97, 547, col. 2, for 1934 read 1936.